Contextualizing the Dynamics of Affective Functioning: Conceptual and Statistical Considerations

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Janne K. Adolf

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Contributions

Substantial portions of this dissertation have been prepared in collaboration with co-authors for publication. I am the primary contributor in all regards.

The work presented in Chapter 3, including a reduced version of Chapter 2, Section 2.2 and excluding Chapter 3, Section 3.6, has already been published as: Adolf, J. K., Voelkle, M. C., Brose, A., & Schmiedek, F. (2017). Capturing Context-Related Change in Emotional Dynamics via Fixed Moderated Time Series Analysis. *Multivariate Behavioral Research*. http://dx.doi.org/10.1080/00273171.2017.1321978.

The work presented in Chapter 4 is in preparation for publication and will be co-authored by Annette Brose, Florian Schmiedek, Manuel C. Voelkle, and Julian D. Karch, who also contributed the proof in Appendix D.

Abstract

Contributions to the recent affect literature highlight the importance of micro-longitudinal studies for understanding daily affective functioning. The resulting intense longitudinal data allow quantifying the temporal regularities that structure short-term (co-)variations of various affective experiences within a person. Via such descriptions of intra-individual *affective dynamics*, one hopes to discover psychological processes underlying affective functioning. Accordingly, dynamic longitudinal modeling techniques get increasingly promoted. In this dissertation, I attempt to address recent calls for a more explicit integration of contextual information into the study of daily affective functioning. Specifically, I focus on modifying popular dynamic models such that they incorporate contextual factors that also fluctuate over time.

In a first contribution, individuals are characterized as being embedded in changing contexts. The proposed approach of *fixed moderated time series analysis* accounts for systemic reactions to contextual changes by estimating change in all parameters of a dynamic time series model conditional upon contextual changes. It thus handles the corresponding problem of intra-individual heterogeneity by treating contextual changes as known and related parameter changes as deterministic. Consequently, model specification and estimation are facilitated and feasible in smaller samples – but information on which and how contextual factors might matter is also required. As a time series model applicable to single individuals, the approach permits the unconstrained exploration of inter-individual differences in contextualized affective dynamics.

In a second contribution, individuals are characterized as interacting reciprocally with their environment. To this end I implement a process perspective on contextual fluctuations by modeling the dynamics of daily events – as a specific contextual factor – using autoregressive models with *Poisson measurement error*. Combining *Poisson and Gaussian autoregressive modeling* offers a possibility to formalize the dynamic interplay between contextual and affective processes, and thereby distinguishes not only unique from joint dynamics, but also dynamics of affective reactivity from dynamics of situation selection, evocation, or anticipation. The models are set up as hierarchical and thus capture inter-individual differences in intra-individual dynamics. Estimation is carried out via simulation-based techniques in the framework of Bayesian statistics.

For both methods, I put emphasis on an accessible and comprehensive presentation. Further, model performance is investigated by simulations under selected finite data conditions, and illustrated by applications to self-report data on affect, stress and daily events from the COGITO study of the Max Planck Institute for Human Development. Finally, both methods are discussed in relation to current micro-longitudinal affect research. Assuming that context matters for understanding daily affective functioning, the methodological considerations put forward in this thesis might support more holistic and differentiated descriptions of daily affective functioning. This might in turn have implications for understanding long-term emotional development and adaption.

Zusammenfassung

Beiträge zur aktuellen Affektliteratur betonen die Bedeutung von Mikrolängsschnittstudien für das Verständnis täglichen affektiven Funktionierens. Die resultierenden intensiven Längsschnittdaten erlauben es, zeitliche Regelmäßigkeiten zu quantifizieren, die kurzfristige (Ko-)Variationen verschiedener affektiver Erfahrungen innerhalb von Personen strukturieren. Über solche Beschreibungen intraindividueller *affektiver Dynamiken* hofft man psychologische Prozesse zu entdecken, die affektivem Funktionieren zugrunde liegen. Dementsprechend werden Verfahren der dynamischen Längsschnittmodellierung zunehmend attraktiv. In dieser Dissertation bemühe ich mich, jüngsten Forderungen nach einer expliziteren Integration kontextueller Informationen in die Untersuchung täglichen affektiven Funktionierens nachzukommen. Im Speziellen konzentriere ich mich darauf, populäre dynamische Modelle so zu modifizieren, dass sie ebenfalls zeitlich schwankende kontextuelle Faktoren einbeziehen.

In einem ersten Beitrag werden Personen als in veränderliche Kontexte eingebettet begriffen. Der vorgeschlagene Ansatz der *festen moderierten Zeitreihenanalyse* berücksichtigt systemische Reaktionen auf kontextuelle Veränderungen, indem Veränderungen in allen Parametern eines dynamischen Zeitreihenmodells auf kontextuelle Veränderungen bedingt schätzt werden. Dem korrespondierenden Problem der intraindividuellen Heterogenität wird also dadurch Rechnung getragen, dass kontextuelle Veränderungen als bekannt und assoziierte Parameterveränderungen als deterministisch behandelt werden. In der Folge sind Modellspezifikation und -schätzung erleichtert und in kleineren Stichproben praktikabel – es sind allerdings auch Informationen darüber erforderlich, welche kontextuellen Faktoren wie eine Rolle spielen könnten. Als Zeitreihenmodell für einzelne Personen erlaubt der Ansatz die uneingeschränkte Exploration interindividueller Unterschiede in kontextualisierten affektiven Dynamiken.

In einem zweiten Beitrag werden Personen als mit ihrer Umgebung wechselseitig interagierend begriffen. Zu diesem Zweck implementiere ich eine Prozessperspektive auf kontextuelle Schwankungen, indem die Dynamiken täglicher Ereignisse – als ein spezieller Kontextfaktor – über autoregressive Modelle mit *Poisson Messfehler* modelliert werden. Die Kombination von *Poisson und Gaußscher autoregressiver Modellierung* ermöglicht es, das dynamische Zusammenspiel kontextueller und affektiver Prozesse zu formalisieren und trennt damit nicht nur zwischen eigenen und geteilten Dynamiken, sondern auch zwischen Dynamiken der affektiven Reaktivität und Dynamiken der Situationsselektion, -evokation oder

-antizipation. Die Modelle sind als hierarchische Modelle aufgesetzt und erfassen so interindividuelle Unterschiede in intraindividuellen Dynamiken. Die Schätzung erfolgt über simulationsbasierte Verfahren im Rahmen Bayesscher Statistik.

Bei beiden Methoden steht eine zugängliche und umfassende Präsentation im Vordergrund. Außerdem untersuche und veranschauliche ich die Modellperformanz durch Simulationen unter ausgewählten Bedingungen endlicher Datenmengen und durch Anwendungen auf Selbstberichtsdaten zu Affekt, Stress und täglichen Ereignissen aus der COGITO-Studie des Max-Planck-Instituts für Bildungsforschung. Schließlich diskutiere ich beide Methoden vor dem Hintergrund der aktuellen mikrolängsschnittlichen Affektforschung. Unter der Annahme, dass Kontext für das Verständnis täglichen affektiven Funktionierens von Bedeutung ist, könnten die in dieser Arbeit vorgestellten methodischen Überlegungen zu ganzheitlicheren und differenzierteren Beschreibungen täglichen affektiven Funktionierens beitragen. Dieses könnte wiederum auf das Verständnis langfristiger emotionaler Entwicklung und Adaptation auswirken.

Contents

Acronyms

AIC	Akaike information criterion
AR	autoregressive
BCI	Bayesian credible interval
CI	confidence interval
CR	cross-regressive
DIC	deviance information criterion
EAP	expected a posteriori
IA	intra-individual
IE	inter-individual
MAP	maximum a posteriori
MCMC	Markov chain Monte Carlo
ML	maximum likelihood
Neff	number of effective samples
PAR	Poisson autoregressive
hPVAR	hybrid Poisson-Gaussian vector autoregressive
RMSE	root-mean-square error
SEM	structural equation modeling
TS	time series
TSA	time series analysis
fmTSA	fixed moderated time series analysis
VAR	vector autoregressive

1 Introduction

Micro- or intense longitudinal assessment protocols, such as daily diaries or experience sampling, permit an ecologically valid, real-time insight into people's daily lives and the varying experiences and behaviors that occur (Ebner-Priemer & Trull, 2009). With their increasing technical feasibility, according research programs get more and more implemented. This development seems especially prominent for research in the domain of affect, where intense repeated measures arise as a standard to assess affective functioning in daily life. The focus of analysis then lies on the temporal regularities that govern the transitions between different affective states within person. These regularities are often referred to as affective *dynamics* and are interesting because they are supposed to reveal the *psychological processes* involved in daily affective functioning, for instance regulation processes (e.g., Boker, 2002; Hamaker, Ceulemans, Grasman, & Tuerlinckx, 2015; Hamaker & Wichers, 2017; Kuppens & Verduyn, 2015; Ram & Gerstorf, 2009). Gaining such a more mechanistic understanding of intra-individual (IA) psychological functioning is of interest per se, but also for explaining the emergence of inter-individual (IE) differences in long-term development. Dynamic models are a relatively natural, self-evident way of parameterizing the temporal dynamics of a single or multiple variables of interest, and are thus potentially helpful in approximating the underlying psychological processes. I shall refer to the rising trend of taking such longitudinal data-driven and dynamic model-driven approaches to affective phenomena as the *micro-longitudinal paradigm* in the following (cf. Hamaker & Wichers, 2017).

Given the increasing popularity of – and remarkable trust in – the micro-longitudinal paradigm in affect research, it seems important to take a closer look at the conditions under which dynamic models in application to longitudinal data do or do not bear psychological substance. The identification of emotion regulation processes hinges on complexities that relate to theoretical, measurement, and modeling decisions. One factor that matters for all such decisions is *context*, that is, the daily environments people live in. Hence, the contextual states which affective experiences and behaviors are *embedded in* and *interact with* matter from a substantive, as well as a statistical perspective.

This work therefore focuses on *contextualized* affective functioning. Specifically, it investigates and develops methodological approaches to incorporate changing contextual states into dynamic models of affective functioning. Ignoring context in dynamic modeling of affective functioning may not only result in biased parameter estimates and flawed conclusions, but also misses the opportunity to investigate contextual effects on affective dynamics, or the interplay of contextual and affective dynamics. While this thesis puts a major emphasis on the methodological-statistical level, the interpretability and supposed psychological substance of the formal models presented remains a criterion.

A *first* line of work concerns *contextual conditions* of affective functioning. Specifically, we (Adolf, Voelkle, Brose, & Schmiedek, 2017) propose an approach to estimating IA change in the parameters of an autoregressive (AR) time series (TS) model, which is a dynamic model typically used to describe affective functioning. The approach, which is called *fixed moderated time series analysis* (fmTSA), examines parameter changes that adhere to a fixed or known shape, and context suggests itself as a factor determining this shape. That is, to the extent it is observed, context (e.g., daily events) can be incorporated as a factor moderating the dynamic model parameters. The model thus represents a parametric solution to the problem of IA *heterogeneity*. In comparison to existing approaches, fmTSA treats heterogeneity as observed, and facilitates model implementation and estimation. Also different forms of change in model parameters can readily be accommodated. We demonstrate the approach's viability given relatively short TS by means of a simulation study. In addition, We present an empirical application to data from the Cognition Ergodicity Study of the Max Planck Institute for Human Development (COGITO study; Schmiedek, Bauer, Lövdén, Brose, & Lindenberger, 2010), targeting the joint dynamics of negative affect and perceived stress and how these are moderated by daily events. I also outline a potential future application of fmTSA to experimental micro-longitudinal data gathered in the context of an emotionally loaded virtual environment (McCall, Hildebrandt, Hartmann, Baczkowski, & Singer, 2016).

While fmTSA may allow for more holistic descriptions of affective functioning in the sense that systemic reactions to contextual variations are taken into account, it is limited in that contextual dynamics themselves are not modelled. In a *second* line of work, focusing on the *contextual complements* of affective functioning, I am therefore concerned with explicit *process models* for contextual changes, specifically, for changes in daily events as a typical instantiation. I make use of flexible modeling techniques, that is, simulation-based model estimation in the Bayesian framework, to account for the characteristic distribution event count data follow (i.e., discrete probability distributions with non-negative support). Here, I adopt and modify a *Poisson autoregressive* (PAR) model (Brandt & Sandler, 2012) that has been developed outside psychology and allows the estimation of dynamic effects at the level of latent event rates. In its modified form, the model quantifies regularities in changes of the rates at which events occur within shorter time periods. Implementing such a process perspective on daily events allows studying contextual dynamics not only on their own, but also in interaction with affective dynamics. I thus incorporate the modified PAR model into a bivariate process model for daily events and affective states, a *hybrid Poisson-Gaussian vector autoregressive* (hPVAR) model. By then simultaneously modeling events and affective experiences as coupled processes over time, I seek to disentangle contextual and intra-personal dynamics. Such statistical effort is critical to justifying the attribution of dynamic effects to intra-personal sources in typically uncontrolled intensive longitudinal data. But it may also yield more differentiated characterizations of the processes involved in daily affective functioning. I again support these modeling suggestions by simulations, and an application to selected data from the COGITO study.

In short, the outline of this thesis is as follows. Chapter 2 provides the wider theoretical and statistical background the present work ties into. To this end, I review and incorporate literature on person-specific psychology, on dynamic models applied to affective phenomena, and on affective dynamics in context. In Chapter 3 and Chapter 4, I present, demonstrate, simulate, apply, and discuss the above introduced approaches of fmTSA and PAR or hPVAR modeling, respectively. Chapter 5 concludes with a general discussion, featuring thoughts on the potentials and limitations of the (contextualized) micro-longitudinal paradigm for understanding affective functioning, adaption and development.

2 The individual as a unit of analysis, dynamic models, and context

This chapter serves to motivate the contributions of this thesis – which are primarily of methodological nature – and features the three main conceptual components underlying this work. I thus present, first, a rationale for the individual person as a unit of analysis, second, a portrait of dynamic models in terms of their statistical properties and their potential psychological substance, and third, different perspectives on the role of contextual factors in affective functioning. I turn to each of the distinct components individually, but also aim to clarify their integration in the context of the present work.

2.1 The individual as a unit of analysis

2.1.1 The problem

Clearly, this work is concerned with psychological processes unfolding within person. Large-sample data on inter-individual (IE) differences, a long-standing standard in psychology, will likely be uninformative for this purpose, as structures of IE variation are usually not identical – not even very similar – to structures of intra-individual (IA) variation. The reason for this is that IE variation also reflects individual levels or means, so, s*table* individual differences, which can per definition not feature in quantifications of IA variation (and vice versa, to some extent).

From the *statistical* viewpoint, one can conceive of this as a special case of Simpson's paradox (for a comprehensive presentation see Kievit, Frankenhuis, Waldorp, & Borsboom, 2013). From the *substantive* viewpoint, it is helpful to recall that one is dealing with potentially very different *sources* of variation at the two levels of analysis (Schmiedek, Lövdén, Von Oertzen, & Lindenberger, 2017; Voelkle, Brose, Schmiedek, & Lindenberger, 2014). Before they may participate in a micro-longitudinal study, individuals generally have a lot of developmental time to accumulate various experiences and behaviors, leading to the emergence of relatively stable IE differences over the long run. When these individuals are then observed for a limited time period with a high temporal resolution, processes of very different nature, operating on top of the established IE differences, might be observed. The somewhat popular typewriter example (cf. Hamaker, 2012) illustrates this well by contrasting *correlated skills* between individuals (i.e., learning experience in typing fast and accurate, accumulated over the long run) with *competing resources* within individuals (i.e., short-term reductions in typing accuracy with increasing typing speed; cf. Von Oertzen, 2014). This line of reasoning is elaborated as “time-as-process” and “time-as-resources” by Ram and colleagues (Ram, Gerstorf, Fauth, Zarit, & Malmberg, 2010, p. 27).

A prominent ambassador for the likely “non-equivalence of structures of IE and IA variation” pertaining to psychological phenomena is Peter Molenaar (n.d., p. 1). His work and the related literature on person-specific methodology (e.g., Browne & Nesselroade, 2005; Cattell, 1952; Hamaker, Dolan, & Molenaar, 2005) provide the fundament for the here-concerned micro-longitudinal affect research that is growing in popularity. For instance, in his widely recognized “Manifesto on Psychology as Idiographic Science”, Molenaar advocates the “epistemological necessity of idiography” for scientific psychology’s progress (P. C. M. Molenaar, 2004, p. 204). Specifically, he calls for a methodology that allows taking into account IA psychological functioning “(...) prior to pooling across other individuals” (P. C. M. Molenaar, 2004, p. 202) in order to identify generalities across individuals and, eventually, general laws that organize human experience and behavior.

This call not only revived a long-standing debate on the role of the individual person in scientific psychology (cf. Lamiell, 1998, 2013). It thus also brought back on the scene the opinion that the analysis of IA variation is somehow incompatible with the generation of so-called “nomothetic” knowledge (cf. Windelband, 1904) drawing upon generalities across individuals. Curran and Wirth, for instance, suggest that a person-centered approach threatens “the systematic building of an empirically based knowledge structure about human development because no knowledge can be generalized beyond the specifics of any single individual” which is then “undermining one of the key goals of empirical science” (Curran & Wirth, 2004, p. 221).

Assuming that quantitative inquiries into processes of psychological functioning have and should have some interest in the generalization of findings from the individual to the population, there are two different lines of argument to address such concerns.

2.1.2 An empirical research agenda

The first line of argument conceives of this as an *empirical* problem (cf. Voelkle et al., 2014). To the extent psychology is interested in IA phenomena, and in the absence of additional information, the individual should be the primary unit of analysis. Only by implementing a person-centered research agenda and using corresponding methodology will it be possible to arrive at "unbiased descriptions and explanations of the differences and commonalities among" IA patterns of variation (Lindenberger & Von Oertzen, 2006, p. 300). Following Molenaar (2004), who does *not* propagate the abolition of IE difference research either, Nesselroade refers to such an empirical agenda as enabling "informed aggregations of information across multiple participants" and concludes, that "a more thorough understanding of the behaviors of interest as they occur at the individual level (...) can usefully inform attempts to articulate lawful relationships that pertain to multiple participants" (Nesselroade, 2010, p. 211). It is thus deemed a *capacity* of a person-centered research agenda – rather than its end in itself – to allow and promote the potentially unconstrained analysis of IA phenomena.

The most forceful implementation of a person-centered research agenda is a *bottom-up* approach in direction from the individual, and individual-specific models, to the population. Such an implementation also embraces the scenario in which no generalities across individuals may be establishable. In response to those in fear of this scenario (e.g., Curran & Wirth, 2004), it may be argued that, when psychological phenomena are characterized by prominent individual differences, "aggregations across individuals must remain on a descriptive, informal level by virtue of the phenomenon under study" (Adolf et al., 2017, p. 23).

Alternatively, one may take more of a *top-down* approach, entailing hierarchical modeling attempts. Hierarchical models afford simultaneous characterizations of IE and IA differences by formalizing individual deviations from some average IA model. With more complex structures of the IA model, such an approach might actually come quite close to implementations of a bottom-up strategy.

The empirical problem-reasoning underlies a large body of intense longitudinal research. For the present thesis, I reanalyze data from a very comprehensive instantiation of this research body, namely the COGITO study (Schmiedek et al., 2010). I implement both a bottom-up approach involving subject-specific models and a top-down approach involving hierarchical models that capture average within-person dynamics.

2.1.3 Some theoretical concerns

Despite the empirical optimism of the "new person-specific paradigm" (P. C. M. Molenaar & Campbell, 2009, p. 112), it is true that a person-specific research agenda does not automatically lead to the successful identification of general laws pertaining to IA psychological phenomena.

Some scholars have raised concerns that the identification of general laws of IA processes may be hampered more fundamentally. The argument made by Borsboom and colleagues

(Borsboom, Kievit, Cervone, & Hood, 2009; see also Borsboom, Mellenbergh, & van Heerden, 2003; Cervone, 2004) is that individual life courses and their causes are so fundamentally different that they can only be compared at an abstract level. The IE difference concepts traditionally used to do so (e.g., extraversion) have arisen as powerful concepts in comparing individuals, exactly because it is in their nature that they "supervene" on IA processes (cf. Borsboom et al., 2009, p. 23 ff.), and thus allow for idiosyncracies in causes, or filter out individual causes[1]. As a direct consequence, however, such constructs themselves lack "causal force" at the IA level and must remain descriptive (Cervone, 2004, p. 184). Process-oriented constructs that possess causal force at the IA level, however, would still have to be proven to be in the same "explanatory league" as IE concepts (Borsboom et al., 2009, p. 27).

The above considerations become relevant and concrete when attempts are made at relating IA patterns of functioning to individual differences in concepts such as global well-being. I elaborate on such attempts in Chapter 2, Section 2.3.3, Chapter 3, Section 3.6, and return to the theoretical concerns in the concluding discussion in Chapter 5, Section 5.2.3.

2.2 *Dynamic modeling of intensive longitudinal affect data*

Given the present focus on the IA level of analysis, dynamic modeling is an attractive or natural – as I have claimed – analysis technique. The reasons for this claim lie in the specific descriptions dynamic models offer for longitudinal data. The following sections include a characterization of the statistical properties typical dynamic models, that is, autoregressive (AR) models, possess, and convey a general rationale for why AR models may match well with models of psychological processes, specifically emotion regulation processes (cf. Adolf et al., in press).

[1]Interestingly, Windelband's (1904) own treatment of nomothetic and idiographic approaches seems to align with this conclusion. He seems to conceive of both approaches as truly complementary in the subjects they target and therefore as not lending themselves to an integration.

2.2.1 Statistical properties of autoregressive models

Dynamic models provide parsimonious descriptions of how people may fluctuate from moment to moment over longer periods. A popular class of dynamic models formalizes change in a quantity of interest as driven by an unobserved stochastic process that is random over time (i.e., a set of random variables that are independent over time; Browne & Nesselroade, 2005). This random process perturbs a process of interest over time, which may then show *non-random, time-structured* change, that is, it may reveal its "intrinsic dynamics" (Boker, 2002, p. 415). In the case of AR models, these *dynamics* take the form of carry-over effects between time points, meaning that the random perturbations to the process of interest propagate over some time. The AR effect then captures the rate at which the process does (not) revert to its long-run mean.

In terms of equations, a univariate AR model of first-order is written as

$$\eta_t = \alpha + \beta\eta_{t-1} + \zeta_t \tag{2.1}$$

with

$$\zeta_t \sim N(0, \psi),$$

where η_t is a continuous-valued stochastic process, the process of interest, α is a regression intercept, β is a lagged regression weight, and ζ_t is a continuous-valued stochastic process that is Gaussian white noise. The random variables $\{\zeta_t\colon t \in 1, \dots, T\}$ are thus normally distributed with zero mean and variance ψ, mutually independent over time, and independent of the process of interest, η_t, at all previous time points.

The process of interest, η_t, and all its constituents may be latent and linked to observable variables via a reflective measurement model of the form

$$Y_t = \tau + \lambda\eta_t + \varepsilon_t \tag{2.2}$$

with

$$\varepsilon_t \sim N(0, \theta),$$

where Y_t is an observed continuous-valued stochastic process, τ is a measurement intercept, λ is a factor loading, and ε_t is a latent Gaussian white noise measurement error process. The random variables $\{\varepsilon_t\colon t \in 1, \dots, T\}$ are thus normally distributed with zero mean and variance θ, mutually independent over time, and independent of the process of interest, η_t, at all previous time points. The measurement intercept and the stochastic measurement residual re66flect measurement error, item-specific and/or time point-specific effects.

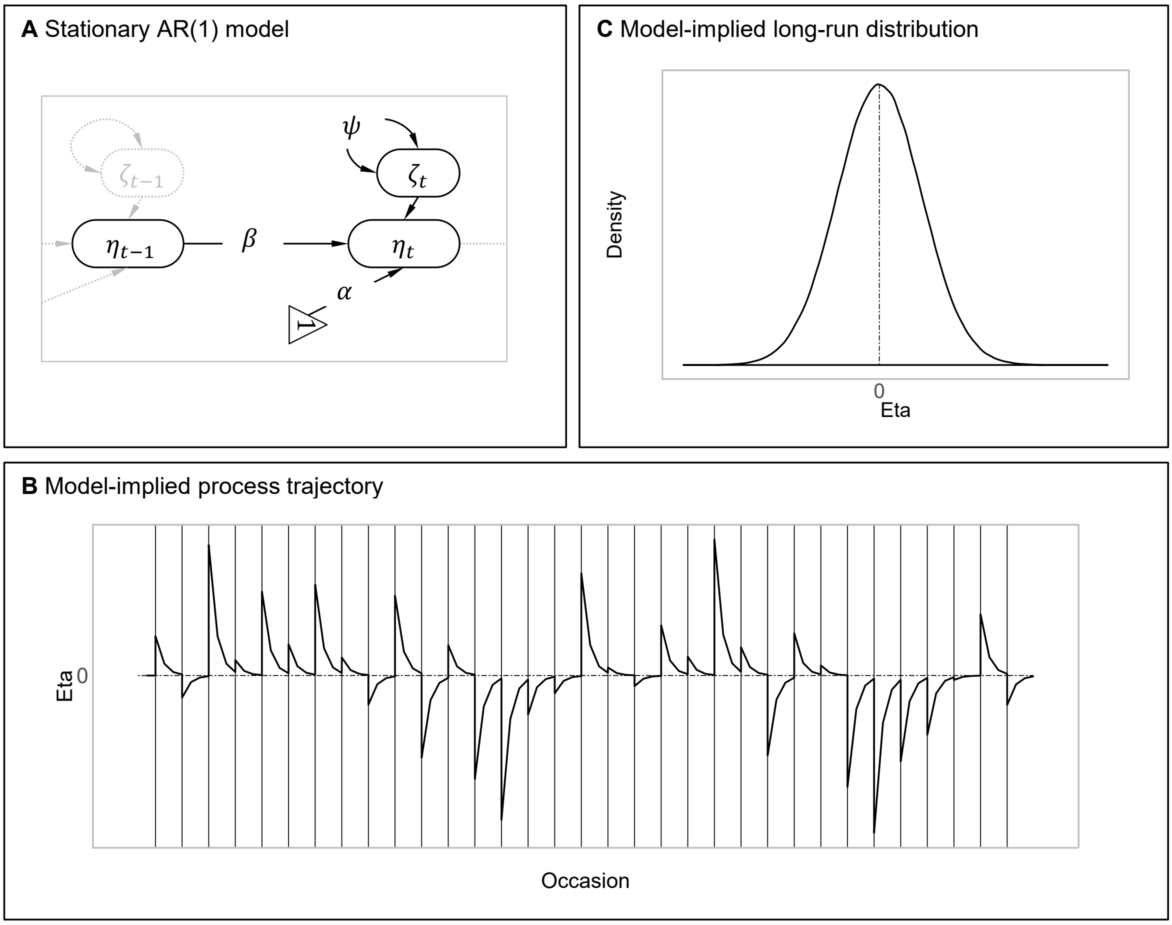


Figure 2.1. Setup and behavior of a stationary AR(1) model. Panel A shows the model in terms of a path diagram. Panel B shows a realization of the model-implied process over time, generated using the following parameter values: $\beta = .3$, $\alpha = 0$, and $\psi = 1$. The bold solid line represents the process of interest, dash-dotted lines show the processes' long-run mean, the occurrence of perturbations is signified by vertical lines in the trajectory plot.

If the latent process η_t is stable (i.e., $|\beta| < 1$) implying that the temporal dependencies are not too strong and perturbations die out in the long run, it has a weakly stationary long-run distribution (i.e., time-invariant mean, variance, and lagged covariances)[2]. For the model under consideration this is a normal distribution with a mean of $\nu = \alpha(1 - \beta)^{-1}$, and a variance of $\rho = \theta(1 - \beta^2)^{-1}$ (Hamilton, 1994, p. 54 ff.).

The behavior of the stationary AR model of first order as presented in Equation (2.1) is visualized in more detail in Figure 2.1. Panel A displays the model in path-diagrammatic terms. The plotted trajectory segment in Panel B demonstrates the temporal patterns or dynamics implied by the model for the process of interest. Specifically, it indicates that a stationary first-

[2] In the context of Gaussian AR models, weak stationarity implies strict stationarity. As far as Gaussian AR models are concerned in the following, I might, for simplicity, just refer to stationarity.

order AR model implies a stable mean level, represented by the dash-dotted horizontal line, from which the process of interest, represented by the bold solid line (e.g., a person's changing affective state), is continuously driven-away by random shocks or perturbations. The perturbations are realizations of the process residual variables over time and may capture all kinds of situational influences (e.g., events). Their occurrence is marked by vertical lines. Following a perturbation to the process of interest, one observes an exponentially shaped return to the mean, the rate of which is inversely related to the strength of the AR effect. That is, the process does not immediately return to its mean. Instead, perturbations "die out" over time (Hamilton, 1994, p. 54), and they do so the faster, the closer the AR effect is to zero. With an AR effect closer to one (and more time-continuous perturbations) one would thus observe prolonged departures from the mean due to the effects of perturbations remaining in the system for longer. In application to affect data, the AR effect has therefore been interpreted as "emotional inertia" (e.g., Brose, Schmiedek, Koval, & Kuppens, 2015; Koval et al., 2015; Koval & Kuppens, 2011; Koval, Kuppens, Allen, & Sheeber, 2012; Kuppens et al., 2012; Kuppens, Allen, & Sheeber, 2010; Suls, Green, & Hillis, 1998), reflecting "regulatory weakness" (De Haan-Rietdijk, Gottman, Bergeman, & Hamaker, 2016, p. 218), the tendency to experience "emotional residues" (Suls et al., 1998, p. 134), and to decouple emotionally from context (Kuppens, Allen, et al., 2010). Panel C of Figure 2.1 shows the model-implied long-run probability distribution of the process. I plot this long-run distribution, as it demonstrates to what extent different states will be covered by the process generally. Hence, both the trajectory segment and the long-run probability distribution provide concrete and complementary descriptions of the kind of data one expects the model to fit well.

The model variant presented so far is a univariate model for the dynamics of a single outcome process. If multiple processes are modeled simultaneously, it is possible to get at the *joint dynamics* among them, that is, how multiple sets of variables affect each other over time. Joint dynamics in terms of cross-lagged effects are of interest, when it comes to questions of lead-lag relationships or "causal dominance" (Schuurman, Ferrer, de Boer-Sonnenschein, & Hamaker, 2016, p. 206; Lövdén, Ghisletta, & Lindenberger, 2005).

Dynamic models may be applied to repeated measures from single cases, here persons, or to repeated measures from multiple cases. For the first instantiation, it is conventional to speak of (dynamic) time series analysis (TSA), where the focus has traditionally been on generalizing over time rather than over cases (Hamilton, 1994). The latter instantiation is conventionally referred to as panel modeling, which usually involves a within- and between-unit (e.g., person) model, hence, a hierarchical model structure (Halaby, 2004).

Finally, dynamic longitudinal models can be contrasted with static longitudinal models. Despite the wide use of the two terms, or at least of the term "dynamic", there does not seem to exist a generally accepted definition. I thus provide the following heuristic distinction. While dynamic models formalize change in the variable of interest in reference to its own past via (stochastic) difference equations or differential equations, and thus take a recursive approach, static models, such as growth curve models, formalize change in the variable of interest as a function of time or some independent time-varying variable (van Geert & Steenbeek, 2005). Static models thus provide descriptions of the overall trajectory of a process, but cannot *directly* address questions of dynamics (Adolf et al., 2017; Hertzog & Nesselroade, 2003; McArdle, 2009; Voelkle, 2016; Voelkle & Oud, 2015).

2.2.2 Autoregressive models as affective process models

Dynamic models, and especially variants of the AR model just portrayed, are increasingly employed in the analysis of intensive longitudinal measures of affective phenomena (e.g., Bringmann et al., 2013; Hamaker et al., 2015; Montpetit, Bergeman, Deboeck, Tiberio, & Boker, 2010; Pe et al., 2015; Röcke & Brose, 2013). The order-one variant is considered a reasonable and parsimonious starting point (Hamaker & Grasman, 2012) and I also rely on the order-one AR model in the remainder of this work.

The aim of these applications is to get at the processes that structure change in daily affective experiences, among them emotion regulation (Boker, 2002; Kuppens, Oravecz, & Tuerlinckx, 2010; Kuppens & Verduyn, 2015; Ram & Gerstorf, 2009). Underlying is the idea that "processes such as psychological adaptation or self-regulation can be extracted from the regularity or order seen across repeated observations over the short term" (Ram & Gerstorf, 2009, p. 782), and, hence, that "a set of intrinsic psychological properties may be able to be extracted from the parameters" (Boker, 2002, p. 405) of dynamic models applied to intense longitudinal data. Formal dynamic models might thus function as affective process models (as in, e.g., Chow, Ram, Boker, Fujita, & Clore, 2005; Kuppens, Oravecz, et al., 2010) and specific model parameters might be interpretable as parameters of regulatory processes. A prominent example is the interpretation of the AR effect as "emotional inertia" (e.g., Kuppens, Allen, et al., 2010), which was introduced in the preceding chapter. This shows how easily dynamic models and their parameters can lend themselves to psychological interpretations.

Indeed, AR-type dynamic models fit well with theoretical models of emotion regulation – possibly due to a shared origin in more general control theoretical notions (Carver & Scheier,

1982) and/or biological models of homeostasis (Bevan, 1965). Prominently featuring in such models is the notion of an affective equilibrium or preferred affective state (Chow et al., 2005; Hess, Kacen, & Kim, 2006; Kuppens, Oravecz, et al., 2010), from which one can be driven away by external influences, but to which one will usually back-regulate. The AR component of the model then captures how quickly this back-regulation or return to the equilibrium happens, or, how long the impact of external influences on the system generally lasts. In addition to a stable mean and a mean-reverting process, a stationary AR model – as any other stationary dynamic model – also implies a stable amount of fluctuations over time. It may therefore be suited more generally to describe everyday life processes and routines that should be somewhat regular and stable over time.

2.3 Contextualizing affective dynamics

Obviously, situational context matters for the study of daily affective functioning, affective dynamics, and emotion regulation. This is not only plausible, but true *per definition* if emotions are conceptualized as quick and ongoing reactions to changing contextual demands that facilitate adaptive behavior (Aldao, 2013; Gross, 1998b; Kuppens, Allen, et al., 2010) or as "the tools by which we appraise experience and prepare to act on situations" (Cole, Martin, & Dennis, 2004, p. 319).

In the following, I recapitulate rationales for putting affective dynamics into context. These include two general substantive perspectives that play fundamental roles in personality and developmental psychology, the *person-situation stance* and the *dynamical systems stance*. Also included is a *statistical rationale* building upon the two substantive motivations. Additionally, I review contributions to the theoretical emotion regulation literature arguing for a contextualization of affective functioning to foster the understanding of its adaptiveness and developmental implications.

2.3.1 The substantive stances: person-situation and dynamical systems

A premise of the so-called person-situation debate in personality psychology (e.g., Epstein & O'Brien, 1985; Kenrick & Funder, 1988) is that "behavior depends on forces inherent in the situation and on forces residing within the person" and the interactions of the two (Krueger, 2009, p. 127; Lewin, 1936). Consequential notions of personality states and variability-based re-conceptualizations of traits (Fleeson, 2001, 2004; Nesselroade, 1988) constitute the very basis for studying states within person, also affective states (Eid & Diener, 1999).

Associated methodological, specifically measurement theoretical work such as latent state-trait theory (Steyer, Ferring, & Schmitt, 1992; Steyer, Mayer, Geiser, & Cole, 2015) capture contextual variations implicitly, via their manifestation in terms of person-states. Additionally, situational effects are defined in complement to stable IE differences. Considering contextual influences more explicitly, and at the IA level of analysis, thus, as providing the conditions for and interacting with intra-personal fluctuations, will be helpful if one wishes to dig deeper into the ways persons and situations interact. Such an emphasis on person-situation interactions within the individual seems to be promoted by procedural perspectives on personality, which hold that "different situational features activate different subsets of units" (Cervone, 2004, p. 186) of an IA personality system. Procedural perspectives on personality thus provide a more concrete motivation for research on affective dynamics – and supposedly related processes of emotion regulation – in context (Cervone, Shadel, Smith, & Fiori, 2006; Kuppens, 2009).

Procedural perspectives on personality also resemble viewpoints promoted in the literature on dynamical systems theory, which is rooted in developmental psychological traditions (e.g., Bergman & Wångby, 2014; Thelen, 2005; van Geert & Steenbeek, 2005). According to dynamical systems theory, individuals are complex dynamical systems, embedded in and interacting with context. That is, individuals are conceptualized "as an organised whole with elements operating together to achieve a functioning system in a dynamic process with interactions between components", with components being "behaviours, biological factors, environmental factors, and so on" (Bergman & Wångby, 2014, p. 31). Drawing upon this very comprehensive conceptualization, it is again implied that context matters for IA functioning and psychological dynamics. As dynamical systems will only reveal their equilibrium-preserving dynamics when not being in equilibrium, so, for instance, when getting perturbed (Boker, 2015), it is straightforward to learn about the system by also looking at perturbations in relation to contextual components.

IA interactions between person and context may take different forms, or may be differently construed. This may range from rather hierarchical notions of individuals and their psychological functioning being *embedded* in situations (Hollenstein, Lichtwarck-Aschoff, & Potworowski, 2013; Ram et al., 2014) to notions of *reciprocal causation*, where both components are complementary parts of a system (Buss, 1977; Ram et al., 2014; Steele, Ferrer, & Nesselroade, 2014). The first notion implies processes at different time scales, so, slower contextual changes that to some extent provide the conditions for faster affective processes. I elaborate on the related possibility to study *systemic reactions* to contextual changes, so, how a system's behavior changes with context in Chapter 3. The second notion imposes less restriction on the temporal setup and concerns questions of coupled process and lead-lag relationships. In this thesis, I present methodological approaches formalizing both notions.

2.3.2 The statistical stance: heterogeneity and spurious dynamics

The statistical stance is concerned with the statistical implications of the substantive perspectives presented above, especially with interpretational fallacies that may occur as a result of statistical solutions that are inappropriate because they do not incorporate context.

Under the *hierarchical* notion of person-context interaction, that is, if affective functioning is embedded in and affected by context, then contextual variations can lead to *IA heterogeneity*. In Chapter 3, Section 3.1.3, I define IA heterogeneity in relation to a stochastic process as the situation, in which more than one set of dynamic parameters is required to characterize the process behavior over time (Adolf et al., 2017). Heterogeneity can produce ambiguous effects and can consequently promote interpretational fallacies (e.g., Simpon's paradox; Wasserman, 2004). In the psychometric literature, this is often discussed with respect to generalizing inferences across levels of analysis, especially the IA and the IE level (e.g., Hamaker, 2012; Kievit et al., 2013), but the issue also arises purely at the IA level (De Haan-Rietdijk, Kuppens, & Hamaker, 2016; Voelkle, 2017). Also, as standard models usually imply homogeneous populations (i.e., independent and identically distributed data), heterogeneity can lead to violations of modeling assumptions which may impact the quality of a statistical modeling solution (Dolan, Jansen, & van der Maas, 2004). Modeling solutions that take instantiations of IA heterogeneity into account will be discussed in Chapter 3, Section 3.1.3 (see also Adolf et al., 2017). An approach that includes context as a observed source of heterogeneity in single-subject TSA constitutes the first major contribution of this thesis.

Under the *reciprocal* notion of person-context interaction, that is, if affective functioning is interacting with context in a reciprocal manner, then temporal patterns in context are an important confounder of temporal patterns of intra-personal source in uncontrolled observational – and thus typical micro-longitudinal – data. That is, as soon as contexts are temporally structured and relevant for the psychological process under study, observed dynamics may be attributable to intra-personal *as well as* contextual dynamics. Then, interpretations of dynamic model parameters as pertaining to characteristics of psychological processes (e.g., "regulatory weakness"; De Haan-Rietdijk, Gottman, et al., 2016, p. 2) are specific attributions of effects that need additional justification. Recent empirical work acknowledges the potential problem of spurious intra-personal dynamics due to temporal characteristics of context (Koval et al., 2015), but process models for contextual changes are not yet used systematically (although there is already work on controlling for contextual differences between individuals, e.g., Brose, Scheibe, & Schmiedek, 2013). A dynamic model to disentangle contextual and affective dynamics and to capture the interplay between contextual and affective changes within person is the second main contribution of this thesis (see Chapter 4). This allows controlling for specific contextual dynamics as a potential confound of affective dynamics in observational data, but also for addressing substantive questions about the reciprocal interplay of contextual and affective processes.

2.3.3 The developmental stance: context and adaption

So far, I have reviewed more abstract perspectives that motivate the study of contextualized affective dynamics within person. To sum up, procedural personality and dynamical systems theories posit that person and context, and specifically IA affective functioning and contextual factors, interact. Taking such interactions into account is necessary to arrive at appropriate descriptions of IA affective variability and IE differences therein, hence, the statistical argument. From a substantive perspective, it will benefit consequent attempts to better understand ongoing emotion regulation.

But arriving at better, potentially more mechanistic models of affective functioning can also address developmental questions. The idea is that, over time, higher-order global emotional states, such as well-being, emerge from the specific affective experiences people make and affective behaviors people show in specific situations in their daily lives. For instance, Larsen speaks of momentary affective states as “components or building-blocks of subjective well-being” (Larsen, 2009, p. 248). Martin and colleagues introduce the concept of “developmental stabilization”, which refers to “the active individual orchestrating multiple subprocesses (…) to achieve stable performance in the higher-order process (…)”, such as well-being (Martin, Jäncke, & Röcke, 2012, p. 186). And finally, complex systems theories borrowed from ecology suggest that continuous drifts in short-term IA affective dynamics may foreshadow abrupt transitions into global pathological states, such as depression (Cramer et al., 2016; van de Leemput et al., 2014; Wichers, Wigman, & Myin-Germeys, 2015).

Individual differences in global emotional states may thus arise from individual differences in the processes of daily affective functioning. Hence, *grounding* global, higher-order emotional outcomes such as psychological health or well-being in the dynamics of daily psychological functioning, is one attempt to *explain* “successful” development (cf. Rowe & Kahn, 1997) and individual differences therein. The hope is thereby to be able to identify *adaptive* patterns that help to, for instance, maintain or increase well-being, and *maladaptive* patterns that fail to maintain or lower well-being (e.g., Kuppens et al., 2012).

The argument in favor of context is now that, if emotion-regulation is context dependent, then contextual information is a crucial complement for understanding such relations. That is, whether certain affective (re-)actions are adaptive, whether ongoing processes contribute to “successful” development, cannot be determined without knowing the contextual conditions. This reasoning is conveyed in recent contributions to the theoretical literature on emotion regulation. Bonanno and Burton note that “(a)lthough stress and coping theory emphasized that coping efficacy was a matter of fit between the strategy and ongoing situational demands, in practice researchers and theorists have tended to catalogue specific coping strategies as either adaptive or maladaptive”, and the authors refer to these categorization as the “fallacy of uniform efficacy” (Bonanno & Burton, 2013, p. 4; see also Aldao, Sheppes, & Gross, 2015). Similar arguments can be constructed from the more data-driven literature on affective functioning. For instance, whereas “emotional inertia” has been described as a “fundamental feature of the emotion dynamics associated with psychological maladjustment” (Kuppens, Allen, et al., 2010, p. 989), being inert or context insensitive in times of sustained high environmental stress is reminiscent of notions of resilience (cf. Montpetit et al., 2010) and might in fact be adaptive (cf. De Haan-Rietdijk, Gottman, et al., 2016).

A strategy to counteract the “fallacy of uniform efficacy” could be to acknowledge and evaluate the contextual conditions a specific emotion regulation strategy, or in the present case, a specific affective pattern is embedded in or related to (Aldao, 2013; Aldao et al., 2015; Bonanno & Burton, 2013). Specifically, including context in assessment and modeling may help to resolve seemingly contradicting findings on the relation of affective dynamics to global emotional outcomes (Koval & Kuppens, 2011; Koval, Pe, Meers, & Kuppens, 2013) and may consequently promote the understanding of notions of “successful” development and long-term adaption. I revisit this specific issue in Chapter 3, Section 3.6 and in the concluding discussion in Chapter 5.

3 Fixed moderated time series analysis

Dynamic models that parameterize temporal regularities in short-term changes are frequently used to cast the dynamics of daily affective functioning. A single dynamic model with time-invariant parameters, however, implies time-invariant affective dynamics, which may not always be sufficient to describe how a person functions over time. In fact, one can think of many factors relating to change in affective dynamics, for instance variations in daily context as individuals switch between work and home, engage in different social interactions or activities, or encounter stressful events (e.g., Koval & Kuppens, 2011; Kuppens, Allen, et al., 2010; Zautra, Berkhof, & Nicolson, 2002).

Here, we (Adolf et al., 2017) propose a dynamic model suited to capture changes in the dynamics of affective functioning via time-varying model parameters. The model is implemented as a time series (TS) model applicable to data from single individuals. Within a given person, the approach allows to freely estimate the amount of change in model parameters that follows a known shape over time. In other words, we consider a model with time-varying parameters, where the change in these parameters is fully explained by an observed variable. This permits testing hypotheses about whether and to what extent differences in observed context are related to differences in affective dynamics or whether and to what extent there are deterministic time trends in affective dynamics. In formal terms, the proposed model addresses the issue of *observed* intra-individual (IA) heterogeneity as opposed to *unobserved* IA heterogeneity. While the sources of heterogeneity are known in the first case, they are unknown in the latter.

The outline of this chapter is as follows. First, we lay out a rationale for applying dynamic time series analysis (TSA) to affective phenomena. We thereby build upon the detailed elaborations in Chapter 2, Section 2.2 and focus here on the TS-quality of the model. We also provide specific arguments in favor of time-varying affective dynamics and recapitulate existing modeling solutions to the corresponding formal problem of time-varying dynamic parameters – or IA heterogeneity – in the $N = 1$ case. Second, we introduce our approach, fixed moderated time series analysis (fmTSA), in terms of model structure as conveyed in equations, in terms of model behavior as illustrated by simulated data, and in terms of parameter estimation. Third, a comprehensive simulation study is presented, in which we investigate the proposed model's performance given relatively short TS (i.e., $T =$

$(100, 150, 200)$). Fourth, we apply the approach to data from the COGITO study (Schmiedek, Bauer, Lövdén, Brose, & Lindenberger, 2010). Using self-report data from nine younger adults across an average of 101 measurement occasions and 132 days, we model the joint daily dynamics of negative affect and perceived stress per subject. Variation in model parameters is estimated as a deterministic function of variation in self-reported daily events. We end by discussing potentials and limitations of the proposed approach.

3.1 Time series analysis applied to affect data

3.1.1 Single subject analyses

In the remainder of this chapter, we are concerned with dynamic models for TS, so, dynamic TSA (e.g., Hamilton, 1994; Harvey, 1989; Lütkepohl, 2005). In Chapter 2, Section 2.2, it has been argued that *dynamic* models yield descriptions of micro-longitudinal affect data that lend themselves well to psychological interpretations, and are therefore frequently used to formalize affective functioning.

The term *TSA* has been used to emphasize that the corresponding models are applicable to data from single individuals (Hamaker & Dolan, 2009; Hamaker et al., 2005; P. C. M. Molenaar, Sinclair, Rovine, Ram, & Corneal, 2009). Single-subject analyses are a necessity in single-case studies, for instance in clinical settings (Roche, Pincus, Rebar, Conroy, & Ram, 2014). But also with repeated measures from many individuals, a *person-centered* or *idiographic* approach can be appropriate if one views it as enabling “informed aggregations of information across multiple participants” (Nesselroade, 2010, p. 211). In concrete terms, TSA solutions from different individuals are independent and subsequent comparisons between individuals thus maximally unconstrained. As argued in Chapter 2, Section 2.1.2, such a bottom-up approach to the scientific goal of establishing generalities across individuals may be considered an “epistemological necessity” when psychological processes are subject to profound inter-individual (IE) differences (P. C. M. Molenaar, 2004, p. 204). The model we present in the following has this capacity.

3.1.2 Rationale for time-varying (affective) dynamics

In many psychological applications, dynamic TS models formalize stationary processes. Stationary processes have stable characteristics over time, specifically, time-stable distributional moments (e.g., mean, variance, auto-covariance), and can thus be cast in terms of time-invariant parameters (Lütkepohl, 2005, p. 24). When thinking of psychological processes in general, and affective processes in particular, a stationary process seems to represent a relatively restrictive case, though. Complex and changing environments as well as ongoing developmental processes likely relate to variability and change in how we function and, thus, how psychological processes evolve over time (Hollenstein et al., 2013; P. C. M. Molenaar, 2004; Nesselroade, 1991).

For instance, an individual's affective dynamics may fluctuate within a certain range (Chow, Zu, Shifren, & Zhang, 2011; Koval & Kuppens, 2011; Ram et al., 2014; Sliwinski, Almeida, Smyth, & Stawski, 2009; Zautra et al., 2002), possibly reflecting "state-dependent regulation" (De Haan-Rietdijk, Gottman, et al., 2016, p. 217). Furthermore, the possibility of temporal trends in the variability and predictability of affect has received interest in the context of forecasting major regime shifts such as transitions into depression (Scheffer et al., 2009; van de Leemput et al., 2014). Finally, major events or enduring changes in a person's environment may result in gradual long-term adjustments in the level around which an individual's affective experiences fluctuate (Boker, 2015).

3.1.3 Modeling solutions to the problem of intra-individual heterogeneity

The processes portrayed in the previous section are all non-stationary in that more than one set of dynamic parameters (i.e., changing dynamic parameters) is required to characterize their behavior over time. We refer to this as a problem of *IA heterogeneity* and thereby take Muthén's (1989) definition of *IE heterogeneity* to the IA level of analysis (Adolf, Schuurman, Borkenau, Borsboom, & Dolan, 2014; Dolan, 2009). Under IA heterogeneity, variation can thus either pertain to a given set of dynamic parameters or to change in dynamic parameters. Clearly, an appropriate modeling approach to explicitly distinguish the two sources is required, if not out of substantive interest then for statistical reasons (e.g., De Haan-Rietdijk, Kuppens, et al., 2016; Dolan et al., 2004).

Traditionally, multivariate modeling solutions to the problem of heterogeneity have been proposed in the context of studying IE differences, encompassing, for instance, multiple-group structural equation modeling (SEM) (e.g., Jöreskog, 1971), SEM with "multiple indicators and multiple causes" (e.g., Muthén, 1989), finite and infinite mixture SEM (Bauer, 2007; Dolan, 2009; Hessen & Dolan, 2009; Lubke & Muthén, 2005; D. Molenaar, 2015; Sterba, 2013), SEM with fixed moderators (e.g., Bauer & Hussong, 2009; Curran et al., 2014; D. Molenaar, Dolan, Wicherts, & van der Maas, 2010), SEM trees and forests (Brandmaier, Prindle, McArdle, & Lindenberger, 2016; Brandmaier, von Oertzen, McArdle, & Lindenberger, 2013), and locally weighted SEM (A. Hildebrandt, Lüdtke, Robitzsch, Sommer, & Wilhelm, 2016; Hülür, Wilhelm, & Robitzsch, 2011). With the rise of person-centered research methods (Hamaker, 2012; P. C. M. Molenaar & Campbell, 2009), problems of IA heterogeneity and according modeling solutions receive increasing attention. In the following, we provide an overview over modeling solutions proposed for the $N = 1$ case. To organize this, we look at two criteria, the major one being whether models address heterogeneity as *observed* versus *unobserved* (cf. Lubke & Muthén, 2005), and the minor, orthogonal criterion being whether change in model parameters is cast in terms of abrupt switches versus smooth trajectories. The first criterion is chosen, because it best contrasts the approach suggested here to existing approaches. The second criterion may be of substantive interest, as the examples in the previous section showed. Obviously, neither is this overview exhaustive, nor are the criteria evoked to structure it.

Treating (sources of) IA heterogeneity as *unobserved* is expedient if one neither knows when nor to what extent the parameters of a dynamic model change, or if the timing and extent of change are at least uncertain. Commonly used modeling solutions address this issue by setting up a parametric probability model for the *change* in parameters. That is, in addition to assuming an unobserved stochastic process leading to changes in the observed outcome variable, these models assume an unobserved stochastic process producing changes in the parameters of the unobserved stochastic process underlying the outcome variable. Fitting such a model involves not only estimating the unknown parameters underlying the observed *outcome* trajectory, but rather estimating the unknown parameters underlying the unobserved *parameter* trajectory underlying the observed outcome trajectory (Kim & Nelson, 1999).

Popular examples are so called regime-switching models (Chow & Zhang, 2013; De Haan-Rietdijk, Gottman, et al., 2016; Dolan et al., 2004; Hamaker & Grasman, 2012; Hamaker, Zhang, & van der Maas, 2009; Hamilton, 2010; Hunter, 2014a; Kim & Nelson, 1999). These evoke a small number of distinct parameter sets (i.e., dynamic regimes) between which the outcome process switches over time according to a discrete-valued parameter process, for instance a Markov chain. Regime-switching models thus allow investigating discontinuous and abrupt changes in model parameters. Other models, on the contrary, evoke a continuous-valued parameter process, for instance a Gaussian autoregressive (AR) process, to account for gradual changes in process parameters over time (e.g., Boker, 2015; Chow, Ferrer, & Nesselroade, 2007; Chow et al., 2011; P. C. M. Molenaar, Beltz, Gates, & Wilson, 2015; P. C. M. Molenaar et al., 2009).

In both cases, discrete- and continuous-valued parameter processes, time-varying covariates may be incorporated into the model to reduce the uncertainty about parameter change. However, covariates are not *necessary* to identify unobserved heterogeneity. This is different in models for *observed* heterogeneity. IA heterogeneity may be treated as observed if the timing of changes in model parameters is assumed to be known. That is, if parameter changes can be coupled to a time-varying covariate or if a temporal pattern of a certain shape can be expected (e.g., a linear trend). In this case, there is no need to estimate the parameter trajectory from the data using a probability model. Instead, parameter change can directly and fully be accounted for by conditioning on the available information on the sources of change in parameters. Only the extent, to which these fixed changes manifest, is then estimated from the data.

Including covariates as fixed moderators has received interest in cross-sectional modeling applications (Bauer & Hussong, 2009; Curran et al., 2014; A. Hildebrandt et al., 2016; D. Molenaar et al., 2010). Here, we propose to transfer the approach to the IA level. We refer to this as *fmTSA*, which can be seen as an extension of vector autoregressive (VAR) models including observed time-varying covariates as fixed or exogenous covariates (e.g., Lütkepohl, 2005, p. 387), which in the standard model only additively affect the outcome variable (but see Bringmann et al., 2013, who let selected parameters of a VAR model vary as a function of observed covariates). In fmTSA, moderators need not be observed, substantive covariates, though. As the data are analyzed in the time domain, model parameters can also vary as a function of time (cf. Bringmann et al., 2016). In fact, the flexibility of readily modeling various shapes of change is one advantage of this model. Other advantages concern an easy implementation and estimation, as we will show in the following. Although it seems unrealistic to have complete information about the timing of parameter changes, incorporating and exploring informed hypotheses can provide a pragmatic starting point for further research. The following sections present the model in more detail.

3.2 *Fixed moderated time series analysis*

3.2.1 Model structure

We employ a time-discrete VAR model of first order as the time-invariant baseline model. The process model is

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \tag{3.1}$$

with

$$\boldsymbol{\zeta}_t \sim N(\mathbf{0}, \boldsymbol{\Psi}),$$

where $\boldsymbol{\eta}_t$ is a $q \times 1$ vector of continuous-valued stochastic processes, $\boldsymbol{\alpha}$ is a $q \times 1$ vector of regression intercepts, $\mathbf{B}$ is a $q \times q$ matrix of auto- and cross-lagged regression weights, and $\boldsymbol{\zeta}_t$ is a $q \times 1$ vector of continuous-valued stochastic residual processes that are Gaussian white noise. The process residual variables $\{\boldsymbol{\zeta}_t : t \in 1, \dots, T\}$ are thus normally distributed with a $q \times 1$ zero mean vector and a $q \times q$ covariance matrix $\boldsymbol{\Psi}$, are mutually independent over time, and independent of the processes of interest, $\boldsymbol{\eta}_{t-1}$, at all previous time points.

If the process is stable (i.e., all eigenvalues of $\mathbf{B}$ have modulus less than 1, implying that the temporal dependencies are not too strong and perturbations die out on the long run), the process variables have a stationary long-run distribution. This is a normal distribution, with mean vector $\boldsymbol{\nu} = (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\alpha}$ and covariance matrix, $\mathbf{P}$, which can only be derived in vectorized form as $vec(\mathbf{P}) = (\mathbf{I} \otimes \mathbf{I} - \mathbf{B} \otimes \mathbf{B})^{-1} vec(\boldsymbol{\Psi})$, where $\mathbf{I}$ is a $q \times q$ identity matrix, $\otimes$ denotes the Kronecker product, and $vec(\boldsymbol{\Psi})$ is the vectorization of $\boldsymbol{\Psi}$.

Since the process variables may be latent, we also specify a reflective measurement model of the form

$$\mathbf{y}_t = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \tag{3.2}$$

with

$$\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Theta}),$$

where $\mathbf{y}_t$ is a $p \times 1$ vector of continuous-valued observed stochastic processes, or indicator processes, $\boldsymbol{\tau}$ is a $p \times 1$ vector of measurement intercepts, $\boldsymbol{\Lambda}$ is a $p \times q$ matrix of factor loadings, and $\boldsymbol{\varepsilon}_t$ is a $p \times 1$ vector of Gaussian white noise measurement residual processes with $p \times 1$ zero mean vector and $p \times p$ diagonal covariance matrix $\boldsymbol{\Theta}$. The measurement residual variables $\{\boldsymbol{\varepsilon}_t : t \in 1, \dots, T\}$ are thus mutually independent over time and independent of the latent residual processes and the latent outcome processes at all previous time points. The model implies that the indicator processes are normally distributed with mean vector $\boldsymbol{\mu} = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\nu}$ and covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\mathbf{P}\boldsymbol{\Lambda}^{\mathrm{T}} + \boldsymbol{\Theta}$, where $\boldsymbol{\Lambda}^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{\Lambda}$.

We now extend the structural model so that all model parameters can vary over time. The model is

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha}_t^* + \mathbf{B}_t^*\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \tag{3.3}$$

with

$$\boldsymbol{\zeta}_t \sim N(\mathbf{0}, \boldsymbol{\Psi}_t^*),$$

and

$$\begin{aligned} \boldsymbol{\alpha}_t^* &= g(X_{t-1}), \\ \mathbf{B}_t^* &= h(X_{t-1}), \\ \boldsymbol{\Psi}_t^* &= i(X_{t-1}). \end{aligned} \tag{3.4}$$

All model parameters can now vary as a function of a time-varying moderator X_t. This moderator enters the model in terms of fixed values and is assumed to completely determine variability in model parameters, that is, $g(\cdot)$, $h(\cdot)$, and $i(\cdot)$ are deterministic functions without stochastic residuals. The form of the functional relationships can be flexibly specified and the extent of covariation with the moderator is freely estimated per parameter. The functions $g(\cdot)$, $h(\cdot)$, and $i(\cdot)$ are themselves time-invariant.

Note that the moderator can be any time-varying variable, including time itself (c.f., Selig, Preacher, & Little, 2012). As the format of the moderator and the form of the functional relationship to the model parameters are (relatively) arbitrary, the model offers reasonable flexibility in testing for different forms of change in model parameters.

For illustrative purposes, we confine ourselves to the minimal example of a bivariate process model in the following, which is

$$\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}_t = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}_t^* + \begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}_t^* \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}_{t-1} + \begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}_t \tag{3.5}$$

with

$$\begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}_t \sim N\left(\begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}_t^*\right),$$

and a linear link function, such that

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}_t^* = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(X)},$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}_t^* = \begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(X)}, \tag{3.6}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}_t^* = \begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(X)}.$$

As in an ordinary regression setup, all model parameters now decompose into an intercept or baseline component for $X_{t-1} = 0$, superscripted by (0), and a change component, associated with a 1-unit change in X_{t-1} and superscripted by (X).

In case of a dummy coded moderator $X_t = x_t \in \{0,1\}$ the model can be re-parameterized as

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(X=1)},$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(X=1)}, \tag{3.7}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(X=1)}.$$

Parameters with superscript $(X = 0)$ still denote the baseline parameters given that $X_{t-1} = 0$. Parameters with superscript $(X = 1)$ now denote the process parameters given that $X_{t-1} = 1$. Instead of estimating change in parameters, the model switches between two distinct processes and estimates their parameters separately. Dependent on one's research question, one may of course choose either of the two mathematically equivalent parameterizations, as long as the moderator is discrete-valued and dummy coded.

Two extensions of the model as presented in Equations (3.5) and (3.6) are possible. First, as in multiple regression, one may include multiple moderators. Second, the measurement model may be specified as time-varying in a similar way as the process model. Like in cross-sectional settings, this may then be used to address measurement theoretical questions (cf. Bauer & Hussong, 2009; Curran et al., 2014; A. Hildebrandt et al., 2016), specifically, it becomes possible to test for violations of factorial invariance over time (e.g., Adolf et al., 2014).

3.2.2 Model behavior

To gain a better understanding of the model's behavior, we present data generated from a univariate variant of the potentially latent process model in the following, thus ignoring the measurement part. Figure 3.1 illustrates the ingredients of this small, exemplary simulation while Figure 3.2 displays the simulated data.

Panel A, Figure 3.1 shows on the left a path diagram of the time-invariant process model. It matches the model conveyed in Equation (3.1), but is reduced to one process, and therefore corresponds to the univariate model presented in Chapter 2, Section 2.2.1. A realization of the model-implied process over time is displayed in the middle part (of Panel A), and the model-implied long-run probability distribution of the process to the right of Panel A. For a detailed description of this time-invariant baseline model's behavior, we would like to refer the reader to the respective Chapter 2, Section 2.2.1. To generate the data, the parameter values are fixed to $\beta = .3$, $\alpha = 0$, and $\psi = 1$.

Panel B of Figure 3.1 displays four time-varying model structures that are based on Panel A's baseline model and differentially include the moderator. To draw the models, we adapted Curran and Bauer's (2007) scheme for path diagrams of hierarchical models. Encircled parameters are not only regressed on a constant of one, with the regression weight reflecting the baseline parameter value if the moderator is zero, but are also regressed on the moderator, and hence are time-varying. Note, however, that, unlike in Curran and Bauer's setup, there are no stochastic residuals pointing to the time-varying parameter variables, indicating that parameter change is not random, but fixed and determined by the moderator. The four models in Panel B entail the following scenarios: A 1-unit increase in the moderator leads to a 0.4-unit increase in the intercept for Model 1, to a 0.5-unit increase in the AR effect for Model 2, to a 0.5-unit increase in the residual variance for Model 3, and to increases in both the intercept (0.4 units) and the AR effect (0.5 units) for Model 4.

For each of these four structures, we consider three formats of the moderator X as visualized via trajectory plots and corresponding long-run frequency distributions in Panel C of Figure 3.1. We incorporate the moderator as an equally distributed dichotomous variable (on the left), as a continuous-valued variable (in the middle) and as a linear function of time (on the right).

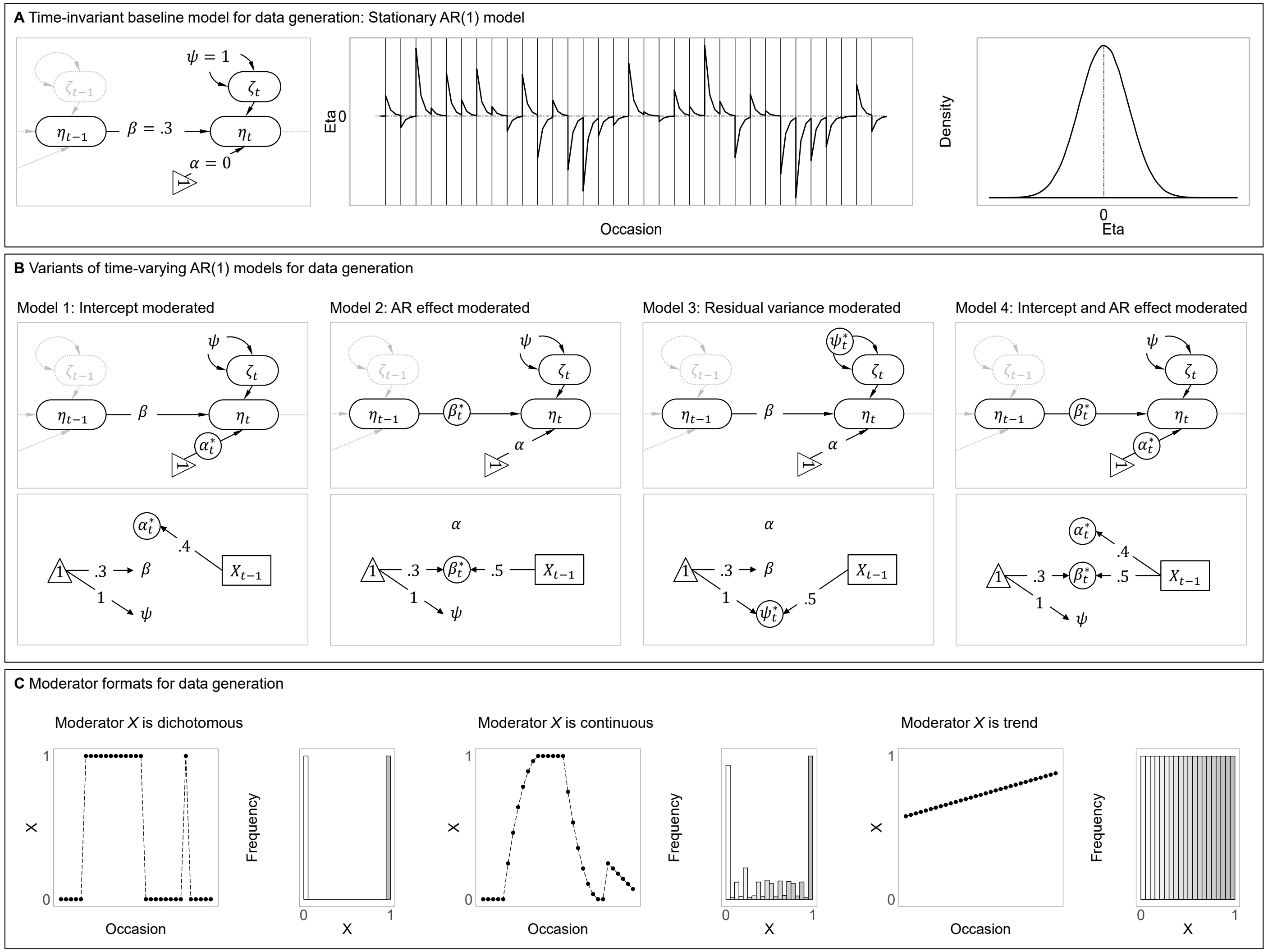


Figure 3.1. Ingredients for a demonstration of the model's behavior. In Panel A, the time-invariant baseline model is displayed in terms of a path-diagram, a segment of a model-implied process trajectory, and the model-implied probability distribution. Data were generated based on four different time-varying model structures, depicted in Panel B, in combination with the three different moderator formats, shown in Panel C. Paths not drawn in path diagrams are zero.

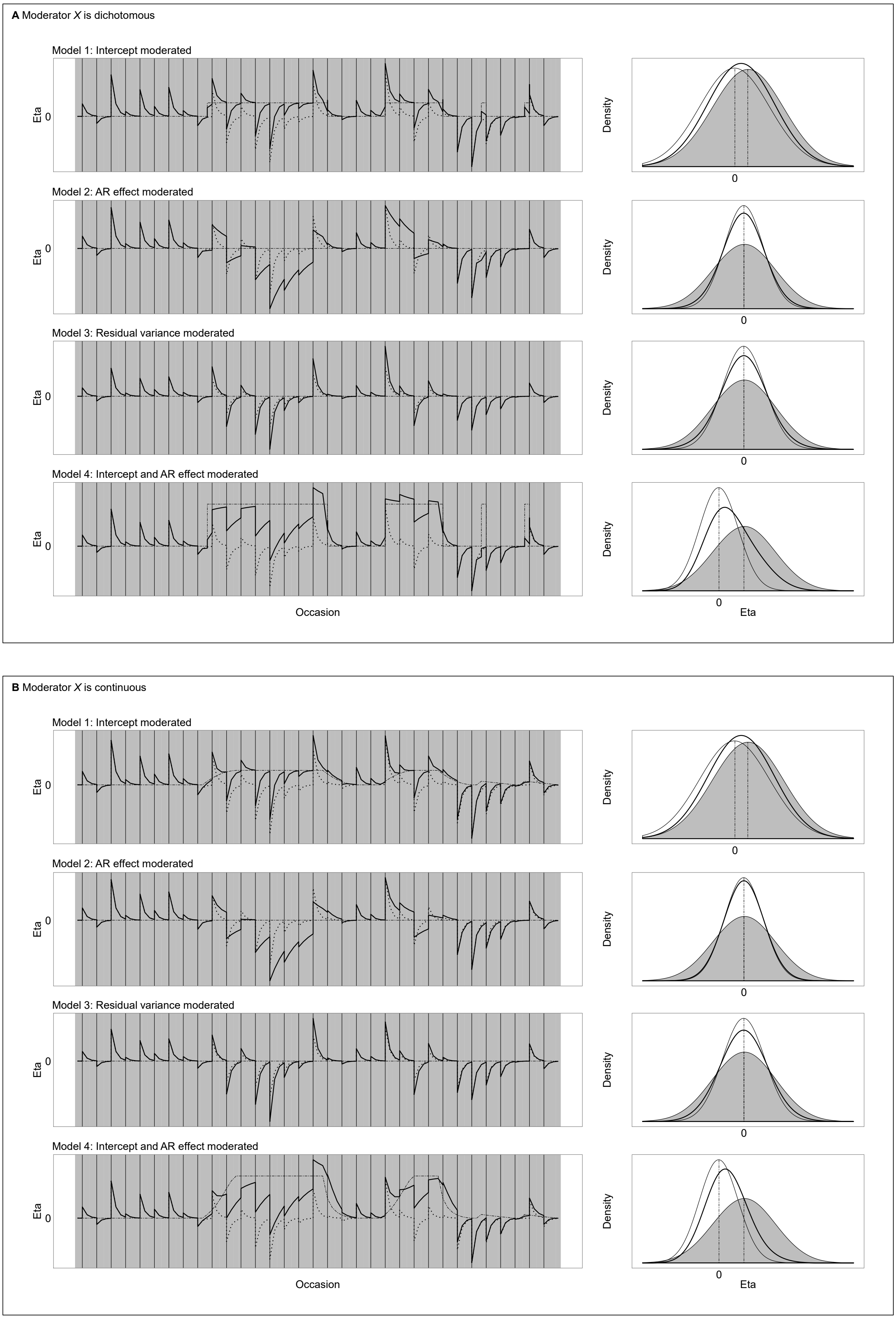
The continuous-valued moderator is a smoothed version of the dichotomous moderator and is obtained by calculating a moving average with a window size of seven occasions (cf. Schilling & Diehl, 2014). We display the formula in the empirical illustration in Chapter 3, Section 3.4, Equation (3.12). For now, it suffices to note that, in contrast to the dummy code, the continuous-valued moderator reflects the accumulation of moderator states over time. Note that the shape of the frequency distribution of this continuous-valued moderator will depend on the temporal stability of the underlying dichotomous moderator. The depicted U-shaped distribution results from a rather stable dichotomous moderator (i.e., the moderator does not switch often between zero and one). Higher instability (i.e., frequent switching) would lead to a concentration of mass in the center of the distribution. The scale of the moderator is in all cases bounded between zero and one and the associated color coding is retained in the following (white indicates zero).

By showing data generated from the combinations of model structures and moderator formats, Figure 3.2 demonstrates what happens when the different simulation ingredients come together. Panels A to C correspond to the different moderator formats and the rows within each Panel to the different model structures. The data are presented in terms of trajectories and long-run probability distributions, as was the case for the baseline model. We present trajectories for the moderated process (bold solid line) and the unmoderated counterfactual (dotted line). The process means conditional on the moderator are also included (dash-dotted line). The values the moderator takes on over time are indicated by background color. We distinguish the probability distribution of the process conditional on the moderator being zero (fine lines, filled white) and one (fine lines, filled grey) from the marginal distribution (in bold) over all values of the moderator. The marginal distribution is always a finite mixture of all conditional distributions, mixed according to the frequency distribution of the moderator, and hence no longer normal.

The trajectory plot in Row 1 of Panel A, Figure 3.2 reveals shifts in mean level due to the intercept shifting with the changing moderator. Note that this reflects a main effect of the earlier moderator state on the current process state after controlling for the effects of earlier process states. The two conditional probability distributions differ only in mean. In Row 2, the AR coefficient and thus the rate at which the process returns to its mean changes and we see how perturbations accumulate more when the moderator is on as compared to when it is off. Remember that the AR coefficient also contributes to the mean. Here, however, we do not see an effect on the mean because the mean is zero in this model. The conditional distributions differ in variance. Row 3 displays a situation in which the strength of a perturbation changes when the moderator is switched on, corresponding to a change in residual variance. Again, the conditional distributions differ in variance. In Row 4, we see the combination of altered mean level (due to changes in the intercept interacting with changes in the AR effect) and altered return rate (due to changes in the AR effect). The conditional distributions differ in mean and variance.

As the continuous-valued moderator is derived from the dichotomous one, Panel B, Figure 3.2 recovers to some extent the just reported pattern of changes in dynamics across models. Differences lie in the smoothness of changes because the moderator changes are smoother. Effects thus build up over time and become more pronounced the longer the moderator is on, before they fade out again.

In combination with a linearly trending moderator (Figure 3.2, Panel C), Model 1 (Row 1) leads to a linear mean change in the moderated process, Model 2 (Row 2) produces monotonically changing return rates, and Model 3 (Row 3) a monotonic change in the strength of perturbations. The combined case of Model 4 (Row 4) is particularly interesting, as the combined changes in the AR coefficient *and* intercept lead to a non-linear mean trajectory. At the same time, the increase in the AR effect causes a decrease in the rate of return to the mean. The moderated process thus diverges from its mean over time.


A Moderator X is dichotomous
Model 1: Intercept moderated
Model 2: AR effect moderated
Model 3: Residual variance moderated
Model 4: Intercept and AR effect moderated
Eta
0
Density
Occasion
Eta
B Moderator X is continuous
Model 1: Intercept moderated
Model 2: AR effect moderated
Model 3: Residual variance moderated
Model 4: Intercept and AR effect moderated
Eta
0
Density
Occasion
Eta

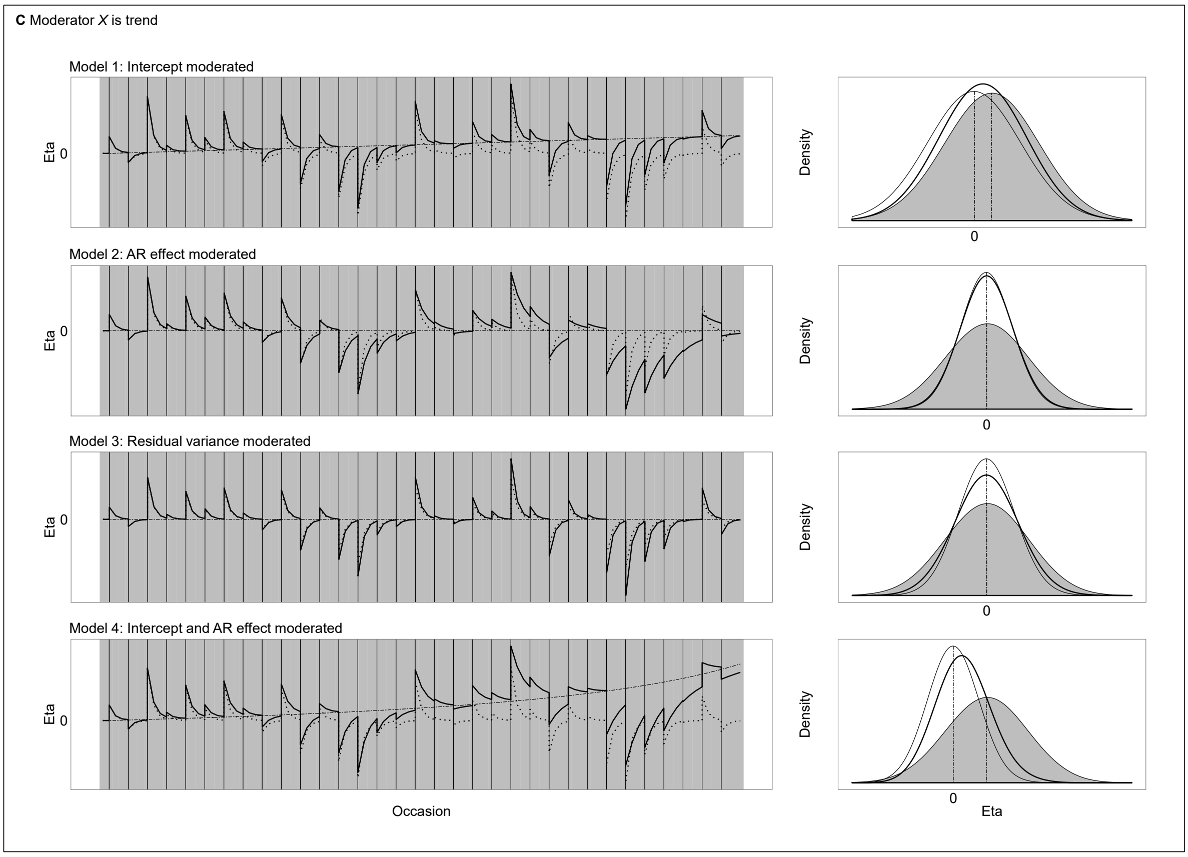


Figure 3.2. Model behavior under different constellations of time-invariant and time-varying parameters (Models 1 - 4) for different moderator formats (Panels A - C). The exact conditions under which the displayed data were generated are visualized in Figure 3.1. Implied process trajectories are shown on the left, implied probability distributions on the right side of the panels. The trajectories of the moderated processes are represented by bold solid lines, the trajectories of the unmoderated counterfactual processes by dotted lines, and the mean trajectories of the moderated processes by dash-dotted lines. Moderator states over time are indicated by background color. The conditional probability distributions are printed with fine contours and in solid white for the moderator being zero and in solid grey for the moderator being one. The marginal probability distribution is depicted with bold contours.

3.2.3 Model estimation

We rely on the state-space modeling framework and the Kalman Filter to estimate the model (e.g., Chow, Ho, Hamaker, & Dolan, 2010; Durbin & Koopman, 2012; Harvey, 1989). State-space modeling evokes a powerful multivariate modeling framework, which distinguishes latent process variables from measurement error. Implementing a dynamic model in state-space modeling requires rewriting the process model in terms of its so called state-space representation, that is, in terms of a first order VAR process (which is possible for all VAR moving average models by extending the latent process vector; e.g., Shumway & Stoffer, 2011). In our case, no reformulation of the model is needed. The Kalman filter is a recursive filtering procedure that capitalizes on this model structure to predict present process states from past ones (e.g., Durbin & Koopman, 2012; Hamilton, 1994; Harvey, 1989). Under the assumption that both the process errors at time t and the process variables at time $t = 0$ (i.e., $\boldsymbol{\eta}_0$) are normally distributed, the Kalman filter yields maximum likelihood (ML) estimates of the model parameters. Usually, for TSA, ML estimation via the Kalman filter is computationally more efficient than ML estimation in the context of SEM (Hamaker, Dolan, & Molenaar, 2003; Voelkle, Oud, von Oertzen, & Lindenberger, 2012).

For the present situation, the joint likelihood function can be written as (cf. Harvey, 1989, pp. 125–128)

$$L(\mathbf{y};\boldsymbol{\Omega}) = \prod_{t=1}^{T} f(\mathbf{y}_t|\mathbf{Y}_{t-1}, X_{t-1};\boldsymbol{\Omega}), \tag{3.8}$$

where

$$\boldsymbol{\Omega} = (\boldsymbol{\tau}, \boldsymbol{\Lambda}, \boldsymbol{\Theta}, \boldsymbol{\alpha}_t^*, \mathbf{B}_t^*, \boldsymbol{\Psi}_t^*),$$

$$\mathbf{Y}_{t-1} = (\mathbf{y}_{t-1}^{\mathrm{T}}, \mathbf{y}_{t-2}^{\mathrm{T}}, \dots, \mathbf{y}_1^{\mathrm{T}}),$$

$$f(\mathbf{y}_t|\mathbf{Y}_{t-1}, X_{t-1};\boldsymbol{\Omega}) = N\big(\mathbf{y}_t\big|\mathbf{Y}_{t-1}, X_{t-1};\boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{t|t-1}, \boldsymbol{\Lambda}\mathbf{P}_{t|t-1}\boldsymbol{\Lambda}^{\mathrm{T}} + \boldsymbol{\Theta}\big),$$

$$\boldsymbol{\eta}_{t|t-1} = \boldsymbol{\alpha}_t^* + \mathbf{B}_t^*\boldsymbol{\eta}_{t-1|t-1},$$

$$\mathbf{P}_{t|t-1} = \mathbf{B}_t^*\mathbf{P}_{t-1|t-1}\mathbf{B}_t^{*^{\mathrm{T}}} + \boldsymbol{\Psi}_t^*.$$

The vector $\boldsymbol{\Omega}$ subsumes all model parameters and $\mathbf{Y}_{t-1}$ contains all observations from time 1 up to time $t-1$. The function $f(\mathbf{y}_t|\mathbf{Y}_{t-1}, X_{t-1};\boldsymbol{\Omega})$ is the distribution function of a multivariate normal with mean vector $\boldsymbol{\mu}_{t|t-1} = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{t|t-1}$ and covariance matrix $\boldsymbol{\Sigma}_{t|t-1} = \boldsymbol{\Lambda}\mathbf{P}_{t|t-1}\boldsymbol{\Lambda}^{\mathrm{T}} + \boldsymbol{\Theta}$. Mean vector $\boldsymbol{\mu}_{t|t-1}$ and covariance matrix $\boldsymbol{\Sigma}_{t|t-1}$ are determined by the model parameters and the process predictions $\boldsymbol{\eta}_{t-1|t-1}$, $\boldsymbol{\eta}_{t|t-1}$, $\mathbf{P}_{t-1|t-1}$, and $\mathbf{P}_{t|t-1}$, which are available from the Kalman filter recursions (for more details see Hamilton, 1994, pp. 377–381; Harvey, 1989, pp. 104–113). Conditioning on $\mathbf{Y}_{t-1}$ eliminates the potential temporal dependencies in the data, that is, it renders the individual likelihoods independent of each other and allows their multiplication to get the joint likelihood.

Conditioning on the moderator X addresses the problem of non-normality due to non-stationarity or IA heterogeneity introduced by X (see the mixture distributions displayed in Figure 3.2). As X is assumed to be observed and can directly be conditioned upon, the time-invariant Kalman filter can be employed. That is, although the extended model in Equations (3.3) and (3.4) formalizes a non-stationary process and implies non-normally distributed data, conditioning on the moderator renders the process stationary and implies normally distributed data. In comparison, models with stochastically time-varying parameters pose more complex estimation problems in that estimation algorithms must be adapted to account for the uncertainty in model parameters (e.g., Kim & Nelson, 1999, pp. 99–102).

The Kalman filter recursions are started conditional on $\boldsymbol{\eta}_{0|0}$, and $\mathbf{P}_{0|0}$. These two quantities are unknown as they concern the latent process state at $t = 0$, a time at which no data are available. They may either be fixed according to the moments of an (un)informative distribution or may, under stationarity, be equated to the earlier introduced long-run mean vector $\boldsymbol{\nu}$ and covariance matrix $\mathbf{P}$ of the latent process and may thus be set up as a function of the model parameters (cf. Harvey, 1989, p. 121). Note, however, that this, in our case, concerns the marginal distribution of the process over X. We derive the marginal distribution of the process for the case of a dichotomous, dummy coded moderator in Appendix A. The moderator at $t = 0$ is assumed to be observed, as for the other time points.

We estimate the model in the free and open source software OpenMx 2.2.4 (Boker et al., 2015; Neale et al., 2015) under R 3.2.1 (R Core Team, 2014). The function *mxExpectationStateSpace* automates the application of the Kalman filter for model estimation by deriving the Kalman filter-based model expectations from the user-provided parameter matrices (Hunter, 2014b). As the model expectations need to be derived conditional on the moderator, we capitalize on the software's flexibility in model specification and incorporate the moderator as a definition variable linking it to all model parameters via the specification of appropriate constraints. An annotated code example is provided in Appendix B.

3.3 *Simulation study*

3.3.1 Purpose and study design

The main purpose of this simulation study is to investigate the performance of the model given relatively short TS (i.e., $T = 100, T = 150, T = 200$).

For data generation, we use the bivariate process model presented in Equations (3.5) and (3.6) and include a dichotomous, dummy coded moderator. We use two sets of parameter values. The parameters of the first set are

$$\begin{aligned}
\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}_t^* &= \begin{bmatrix}2\\ 3\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}2\\ -1.5\end{bmatrix}^{(X)},\\
\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}_t^* &= \begin{bmatrix}.3 & -.15\\ -.15 & .3\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}.3 & -.1\\ -.1 & .3\end{bmatrix}^{(X)},\\
\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}_t^* &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}.5 & 0\\ 0 & .5\end{bmatrix}^{(\mathrm{X})}
\end{aligned} \tag{3.9}$$

under the change-parameterization (cf. Equation (3.6)), and

$$\begin{aligned}
\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}_t^* &= (1 - X_{t-1})\begin{bmatrix}2\\ 3\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}4\\ 1.5\end{bmatrix}^{(X=1)},\\
\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}_t^* &= (1 - X_{t-1})\begin{bmatrix}.3 & -.15\\ -.15 & .3\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}.6 & -.25\\ -.25 & .6\end{bmatrix}^{(X=1)},\\
\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}_t^* &= (1 - X_{t-1})\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}1.5 & 0\\ 0 & 1.5\end{bmatrix}^{(X=1)}
\end{aligned} \tag{3.10}$$

under the switch-parameterization (cf. Equation (3.7)). Here it can be seen that we chose values such that the two conditional processes ($(X = 1)$ and $(X = 0)$) each have a mirror symmetric **B** matrix containing the AR and cross-regressive (CR) effects. Although this symmetry is neither necessary nor realistic for most practical situations, it is useful here as it enables an easier detection of finite sample biases in the following, which partly depend on the size of the estimated effects.

We additionally look at second set of parameters within this simulation which are

$$\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}0\\5\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}3\\2\end{bmatrix}^{(X=1)},$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\\beta_{21} & \beta_{22}\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}.5 & .4\\-.3 & .7\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}.3 & .5\\0 & .8\end{bmatrix}^{(X=1)}, \tag{3.11}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\\psi_{21} & \psi_{22}\end{bmatrix}_t^* = (1 - X_{t-1})\begin{bmatrix}1 & 0\\0 & 1.2\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}1.5 & 0\\0 & 1\end{bmatrix}^{(X=1)}$$

under the switch-parameterization. These values are taken from a regime switching model featuring in a recent paper (Hamaker & Grasman, 2012) that investigates model performance in small samples with missing data. We reuse the parameter values here to enable a performance comparison between the regime-switching model and our approach. The comparison grounds on the fact that the two models imply comparable parameter and outcome processes. The comparison is *limited* in that the models themselves approach heterogeneity in very different ways. As discussed earlier, the regime-switching model freely estimates the process of switching between dynamic regimes from the data and thus accommodates unobserved IA heterogeneity. Our approach assumes that switching times between dynamic regimes are known, accommodating observed IA heterogeneity.

We combine the process model variants displayed in Equations (3.9) to (3.11) with a minimal version of the measurement model, in which we specify a one-to-one relationship between manifest and latent variables by fixing $\mathbf{\Lambda}$ to an identity matrix and the measurement residual variances to zero.

The frequency distribution of the moderator is varied such that it is present (i.e., taking on the value 1) in 20, 30,…, 80 % of the occasions. The cut-offs of 20 % and 80 % imply that model estimation is supported by at least 20 observations per moderator state, which prevents us from larger numbers of non-converging solutions. The first major part of the simulation (Chapter 3, Sections 3.3.2 and 3.3.3) is based on moderator occurrences that are random in time. In an additional part (Chapter 3, Section 3.3.3), we complement this by looking at time-structured moderator changes and how this affects performance of the model.

We present results in terms of biases of point estimates and coverage rates of the 95% confidence interval (CI) estimates. Relative parameter bias in % of the generating parameter value is reported if the generating parameter is different from zero, absolute bias otherwise. We rely on profile likelihood-based CI estimates as implemented in OpenMx. Unlike Wald-type CIs that only approximate the shape of the likelihood function around the maximum, likelihood-based interval estimates use the exact shape of the likelihood function, and are preferable with small sample sizes (Pek & Wu, 2015). If parameter correlations are reported, they are drawn from the averaged transformed Hessian matrix but can also be recovered empirically across replications.

3.3.2 Performance in small samples

Table 3.1 and Table 3.3 show parameter biases and CI coverage rates per moderator frequency condition for the first parameter set under the switch-parameterization (Equation (3.10)).

In general, we observe problems of finite sample bias which are more pronounced the shorter the overall TS. These biases are only specific to our model in the sense that, across moderator frequency conditions, we in principle compare models fitted to TS of varying length. Overall, bias is larger the shorter the conditional TS. That is, parameters with superscript $(X = 0)$ show increasing bias with increasing moderator frequency, whereas parameters with superscript $(X = 1)$ show decreasing bias with increasing moderator frequency. These trends are most obvious for the residual variances, but are also present for the AR and CR parameters (although this is sometimes obscured by compensating effects between the parameters), and for the intercepts. Here, the patterns of bias can best be seen for effects of bigger size, hence, the parameters with superscript $(X = 1)$. By plugging the mean estimates across replications into the model-implied expressions of the long-run mean vector and covariance matrix (Appendix A), we learn that, per conditional process, the model-implied means are in relative consistence with the generating means. Thus, the biases in intercepts likely compensate for biases in the AR and CR effects as these highly correlated parameters sets both contribute to the means. Indeed, fixing all AR and CR effects to their true values removes the biases in intercepts. The model-implied variances and covariances of the conditional processes, on the contrary, are too low in comparison to the generating ones. Underestimation of the variance given an unknown mean is a well-known problem in ML estimation (Bishop, 2006). In this model, the conditional variances and covariances are a function of the AR and CR coefficients and the residual variances. Accordingly, fixing the intercepts, the AR, and CR effect (i.e., rendering the means known) removes the biases in residual variances. However, fixing the intercepts only (i.e., reducing the degrees of freedom in the estimation of the means) reduces bias in the AR and CR effects, but does not remove it. In fact, it has been shown that estimates of AR (and possibly CR) effects are biased towards zero even with a known mean (Cheang & Reinsel, 2000; Marriott & Pope, 1954).

In sum, we observe three instantiations of finite sample bias. One concerns underestimation of the variance given an unknown mean, the second concerns bias towards zero in the AR (and CR) effects, and the third concerns compensatory bias in the intercepts. Note that, as displayed in Table 3.2, the very same biases show up in a different pattern if we directly estimate change in model parameters via the change-parameterization. The effects are again most pronounced for the residual variances, for which change is underestimated for low moderator frequencies and overestimated for high moderator frequencies. This is due to the way the above biases add up under this parameterization. Revisiting Table 3.1, we recall, that we underestimate the baseline parameter much less than the alternative state parameter for low moderator frequencies under the switch-parameterization. Because the alternative state parameter is the higher one in the generating model, we see an underestimation of the difference between the two processes. For low moderator frequencies, on the contrary, we underestimate the lower baseline parameter more than the higher alternative state parameter, resulting in an overestimation of the difference.

Our recommendation is to rely on CI estimates rather than point estimates in small samples. Although they mirror the pattern of biases described, the coverage rates of the profile likelihood-based CIs seem acceptable even for $T = 100$ – at least if the frequency distribution of the moderator is relatively symmetric (i.e., with a mean of around .5).

Table 3.1. Model performance (switch-parameterization) in terms of relative biases of point estimates in %

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
									$T = 200$									
.2	1.00	1.21	-1.38	1.47	1.17	-2.86	-1.49	0.00	-1.71	0.91	2.52	-1.87	-0.80	0.81	-2.12	-6.90	-0.01	-7.55
.3	1.13	0.78	-1.21	0.02	1.59	-2.64	-1.55	0.00	-1.99	0.70	4.04	-1.11	1.24	1.31	-2.60	-4.73	0.00	-5.31
.4	0.79	0.99	-0.61	1.68	1.34	-2.59	-1.68	0.00	-2.24	0.77	4.07	-1.14	1.40	0.90	-2.74	-3.81	0.00	-3.71
.5	0.44	0.74	-0.09	1.21	0.78	-1.89	-1.94	0.00	-2.56	0.86	3.95	-0.99	1.62	1.60	-2.63	-2.87	0.00	-2.65
.6	0.98	0.92	-0.40	1.71	3.05	-2.37	-2.60	0.00	-3.63	0.92	3.15	-1.02	1.71	1.19	-2.22	-2.32	0.00	-2.46
.7	0.30	0.73	0.12	1.91	1.17	-1.91	-3.77	0.00	-4.29	1.01	3.55	-1.04	2.25	1.08	-2.38	-2.08	0.00	-2.05
.8	-0.65	0.76	0.73	2.00	-0.38	-2.07	-6.20	0.00	-7.15	1.36	2.64	-1.17	2.00	1.38	-2.12	-1.86	0.00	-1.98
									$T = 150$									
.2	1.18	1.76	-1.89	1.72	0.81	-4.47	-1.66	0.00	-2.33	0.82	6.93	-1.62	1.83	1.49	-4.17	-9.77	0.00	-9.99
.3	1.22	1.59	-1.49	1.73	1.16	-3.91	-1.92	0.00	-2.94	0.49	5.65	-1.11	1.02	-0.09	-4.12	-6.44	0.00	-7.33
.4	1.07	0.92	-0.55	1.19	2.27	-2.64	-2.50	0.00	-2.94	1.13	4.84	-1.52	1.39	1.87	-3.63	-5.08	0.00	-5.35
.5	0.56	1.01	-0.30	1.63	1.29	-2.98	-3.07	0.00	-3.75	0.73	4.54	-1.00	1.95	0.75	-3.13	-3.61	0.00	-4.07
.6	0.35	1.23	0.05	2.34	0.50	-2.98	-3.67	0.00	-4.68	1.12	4.80	-1.24	2.68	1.28	-3.29	-3.19	0.00	-3.62
.7	0.02	0.79	0.28	1.89	1.44	-1.60	-5.29	0.00	-6.12	1.14	5.05	-1.22	3.40	0.80	-3.36	-2.55	0.00	-3.04
.8	1.21	1.14	-0.52	3.12	4.99	-2.54	-7.74	0.00	-9.66	1.45	3.51	-1.29	2.62	0.59	-2.84	-2.56	0.00	-2.65
									$T = 100$									
.2	0.98	2.42	-2.52	1.52	-1.02	-6.54	-2.96	0.00	-3.68	0.95	9.37	-2.09	2.68	1.64	-5.80	-13.72	0.01	-15.12
.3	1.34	2.72	-1.83	3.97	1.28	-6.55	-3.28	0.00	-4.40	1.58	8.95	-2.24	2.33	3.41	-5.98	-9.55	0.00	-9.95
.4	1.62	2.10	-1.50	3.27	2.08	-5.47	-3.14	0.00	-4.74	1.70	8.71	-2.46	3.30	2.89	-5.90	-7.34	0.00	-7.36
.5	1.32	1.82	-0.61	2.79	3.30	-4.70	-4.52	0.00	-6.08	1.56	7.06	-1.95	3.00	2.56	-5.03	-5.56	0.01	-6.14
.6	1.09	1.97	-0.65	4.07	1.95	-4.69	-4.95	0.00	-6.54	1.53	6.74	-1.86	3.53	1.87	-4.71	-4.67	0.00	-5.19
.7	0.98	1.64	-0.13	3.93	2.95	-3.95	-7.39	0.00	-9.31	2.01	6.20	-2.01	4.05	2.15	-4.67	-4.17	0.00	-4.39
.8	0.31	1.42	-0.07	3.63	4.72	-4.14	-12.04	0.00	-14.39	2.55	5.62	-2.25	4.32	2.57	-4.63	-3.36	0.00	-3.68

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being on in relative numbers of occasions, Equation (3.10) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

Table 3.2. Model performance (change-parameterization) in terms of relative biases of point estimates and coverage rates of 95 % interval estimates in %

Freq X	Relative biases									Coverage rates								
	$\alpha_1^{(X)}$	$\alpha_2^{(X)}$	$\beta_{11}^{(X)}$	$\beta_{21}^{(X)}$	$\beta_{12}^{(X)}$	$\beta_{22}^{(X)}$	$\psi_{11}^{(X)}$	$\psi_{21}^{(X)}$	$\psi_{22}^{(X)}$	$\alpha_1^{(X)}$	$\alpha_2^{(X)}$	$\beta_{11}^{(X)}$	$\beta_{21}^{(X)}$	$\beta_{12}^{(X)}$	$\beta_{22}^{(X)}$	$\psi_{11}^{(X)}$	$\psi_{21}^{(X)}$	$\psi_{22}^{(X)}$
	$T = 200$																	
.2	0.72	-3.07	-1.35	1.43	2.78	-4.05	-18.07	0.00	-19.09	93.80	94.02	94.00	94.13	94.00	94.32	93.82	93.87	92.75
.3	0.06	-1.87	-1.01	1.19	-0.89	-2.49	-11.22	0.01	-11.92	94.78	94.53	94.43	94.25	94.23	95.00	93.95	94.28	93.62
.4	0.72	-1.41	-1.65	-0.95	1.99	-2.70	-6.26	0.01	-5.23	94.47	94.57	94.40	94.63	94.42	94.37	94.87	94.22	94.33
.5	1.33	-0.87	-2.21	-0.97	1.77	-2.40	-3.83	0.00	-4.32	94.25	95.15	94.05	95.05	94.67	94.65	94.87	94.80	94.75
.6	0.70	-1.68	-1.39	1.16	-2.94	-3.04	-1.08	0.00	-0.61	94.42	94.62	94.88	94.53	94.53	94.75	95.00	94.98	94.85
.7	2.00	-1.44	-2.33	2.37	0.59	-2.22	1.87	0.00	3.90	94.40	94.52	94.30	94.20	94.50	94.82	94.68	94.52	94.25
.8	3.19	-0.98	-2.98	2.23	2.39	-2.36	4.97	0.00	9.21	93.93	94.02	93.90	94.17	94.17	94.23	93.88	94.87	93.82
	$T = 150$																	
.2	0.95	-2.63	-1.13	1.71	5.07	-2.88	-24.13	0.00	-24.82	93.52	93.98	93.78	94.08	92.93	94.27	92.33	93.42	92.80
.3	1.53	-1.98	-2.58	-2.56	3.73	-4.06	-17.11	0.00	-14.55	93.80	94.85	94.07	94.23	93.95	94.55	93.67	94.53	93.68
.4	0.34	-1.96	-1.50	0.59	-1.98	-3.01	-8.82	0.00	-8.58	94.33	94.60	94.03	94.52	93.95	94.77	93.97	94.52	94.43
.5	1.05	-2.55	-2.58	1.94	-0.48	-4.22	-6.11	0.00	-5.13	94.27	94.00	94.23	94.62	94.32	94.48	93.87	94.32	94.13
.6	0.94	-2.43	-1.66	3.30	-2.04	-3.65	-1.86	0.00	-1.00	94.33	94.47	94.13	94.38	94.35	94.25	94.60	94.55	94.37
.7	2.87	-2.32	-3.34	2.82	4.10	-4.17	3.28	0.00	3.08	94.63	94.38	94.22	94.10	94.55	94.00	93.68	93.70	94.25
.8	2.58	-1.28	-2.46	2.12	-4.13	-3.07	7.61	0.00	11.29	93.82	93.98	93.83	94.12	94.42	93.97	94.02	93.80	93.68
	$T = 100$																	
.2	-0.79	-1.91	-0.56	0.68	-1.66	-3.43	-36.33	0.00	-39.38	93.12	92.75	92.80	92.75	92.88	93.05	83.32	91.92	83.25
.3	-0.33	-5.04	-1.74	1.74	-3.40	-7.54	-22.29	0.01	-22.73	93.38	93.33	93.32	93.68	93.83	93.15	92.47	93.60	92.17
.4	0.73	-4.04	-2.33	2.64	2.43	-5.95	-14.49	0.01	-15.23	94.27	94.37	94.25	93.93	94.27	94.55	93.90	93.93	93.62
.5	1.95	-3.89	-3.69	4.32	2.29	-5.36	-9.07	0.01	-6.42	94.10	94.42	93.65	94.12	94.15	94.53	93.47	94.08	94.25
.6	2.61	-4.09	-3.70	4.60	1.19	-5.89	-3.05	0.01	-0.93	93.72	94.25	93.97	94.42	93.77	94.62	93.90	94.37	94.88
.7	4.70	-0.86	-5.25	0.00	3.42	-4.58	2.53	0.00	6.42	93.85	94.05	93.60	93.58	93.97	93.97	94.20	94.17	94.28
.8	3.71	-0.85	-3.72	1.95	-3.25	-3.90	13.41	0.00	15.74	92.98	93.35	92.90	93.22	92.87	92.98	90.50	92.70	92.30

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being on in relative numbers of occasions, Equation (3.9) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

Table 3.3. Model performance (switch-parameterization) in terms of coverage rates of 95 % interval estimates in %

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
									$T = 200$									
.2	94.92	94.97	94.25	94.93	94.75	94.25	94.12	94.70	94.17	93.53	94.13	93.27	93.77	93.52	93.52	92.50	93.60	92.77
.3	94.73	95.03	94.43	95.27	95.03	95.07	94.40	94.52	93.87	94.73	94.98	94.57	94.73	94.55	94.53	93.47	94.42	92.72
.4	94.48	94.55	94.45	94.65	94.38	94.60	94.23	94.47	94.28	94.50	94.50	94.40	94.57	94.92	94.78	93.77	94.55	94.27
.5	94.65	94.77	94.80	94.72	94.63	94.78	94.05	94.45	94.10	94.38	94.83	94.50	94.55	94.83	94.57	94.10	94.47	94.40
.6	94.35	94.50	93.98	94.13	94.35	94.15	94.00	94.38	94.07	94.55	94.70	94.33	94.78	94.57	94.37	94.57	95.13	94.77
.7	94.25	94.65	94.40	94.63	94.35	94.38	93.72	93.67	93.45	94.90	95.12	94.87	95.10	94.40	95.22	94.67	94.98	94.63
.8	93.95	94.10	93.63	94.25	93.80	94.13	92.48	93.57	93.00	94.62	95.00	94.80	94.58	94.80	94.68	94.68	94.32	94.87
									$T = 150$									
.2	94.72	95.02	94.87	94.55	94.90	94.52	94.30	94.40	94.30	93.33	93.05	93.77	93.67	93.48	93.60	92.42	93.18	91.58
.3	95.12	94.23	94.80	94.77	95.07	94.43	94.20	94.55	93.92	94.28	93.98	93.65	93.60	94.18	93.48	93.50	94.02	92.85
.4	94.58	94.47	94.62	94.67	94.83	93.88	94.43	93.77	94.25	94.02	94.68	94.08	94.45	94.38	94.33	94.07	93.93	93.42
.5	94.27	94.93	94.22	94.22	94.15	95.02	94.03	93.55	94.07	94.65	94.20	93.90	94.42	95.05	94.57	93.90	93.85	94.17
.6	94.30	94.13	94.38	93.92	94.75	94.27	93.15	94.37	94.30	94.83	94.28	94.42	94.63	94.53	94.13	94.55	94.40	93.20
.7	93.62	94.60	93.17	94.30	93.62	94.03	93.00	93.62	93.25	94.22	94.62	94.15	94.48	94.48	94.75	94.97	94.80	93.53
.8	93.35	93.88	93.28	93.80	93.73	93.58	91.63	92.70	90.95	94.63	94.10	94.87	94.30	94.58	94.68	94.43	94.95	94.27
									$T = 100$									
.2	95.05	94.23	94.85	94.13	94.73	94.55	93.87	94.33	94.23	92.58	92.65	92.43	92.27	92.85	92.78	82.13	92.12	82.02
.3	94.30	93.85	94.32	93.70	94.42	93.82	93.60	93.82	93.68	93.33	93.52	93.23	92.98	93.32	93.30	92.17	93.63	92.03
.4	94.62	93.85	94.67	93.97	94.10	93.93	93.97	94.38	93.82	94.23	93.72	93.60	93.58	94.20	94.15	92.80	93.72	93.13
.5	94.05	94.23	93.67	93.85	94.07	94.37	93.70	93.25	92.85	94.65	94.30	94.48	94.32	94.15	94.00	93.48	94.40	92.75
.6	93.43	93.92	93.52	94.18	94.32	94.40	93.47	93.23	92.33	94.22	94.18	94.25	93.97	94.38	94.27	93.62	94.43	93.95
.7	93.00	93.53	93.20	93.25	93.20	93.60	92.05	92.95	91.92	94.87	94.53	94.33	94.43	94.72	94.35	94.03	94.12	93.27
.8	92.53	92.80	92.28	92.68	92.98	93.20	89.52	91.02	90.52	93.95	94.53	94.07	94.57	94.73	93.68	94.12	94.33	94.10

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being on in relative numbers of occasions, Equation (3.10) contains the generating model.

Table 3.4. Model performance (switch-parameterization) in terms of relative biases of point estimates and coverage rates of 95 % interval estimates in %

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
	Relative biases																	
.6	0.00	3.93	-1.71	0.00	2.90	-3.27	-7.19	0.00	-7.46	1.04	11.97	-4.87	0.00	2.34	-2.97	-4.91	0.00	-4.80
	Coverage rates																	
.6	94.15	93.80	93.92	93.87	94.52	93.72	92.33	93.50	92.58	94.00	93.82	94.43	94.13	94.40	93.97	93.53	94.35	93.68

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being on in relative numbers of occasions, Equation (3.11) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

Table 3.5. Model performance (switch-parameterization) for different moderator change patterns and for a time-invariant reference model in terms of relative biases of point estimates in %

Change X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
random	1.32	2.36	-6.65	-1.96	-2.48	-6.60	-6.24	0.00	-6.20	1.74	2.62	-7.65	-2.27	-1.73	-7.36	-7.01	0.00	-6.17
Slow	4.08	5.42	-14.68	-2.85	-1.92	-14.65	-5.97	0.01	-5.96	3.04	4.98	-15.23	-4.10	-5.29	-14.10	-5.94	0.00	-5.94
Fast	-0.49	0.80	-0.51	-4.11	-2.48	-3.03	-6.35	0.00	-6.42	0.23	0.14	-2.35	-4.87	-1.85	-1.81	-6.31	0.00	-5.81
No X	α_1	α_2	β_{11}	β_{21}	β_{12}	β_{22}	ψ_{11}	ψ_{21}	ψ_{22}	-	-	-	-	-	-	-	-	-
$T = 50$	3.99	5.23	-15.29	-2.89	-2.17	-14.41	-6.14	0.00	-6.59	-	-	-	-	-	-	-	-	-

Note. The number of replications is 6,000 per line for the time-varying conditions including X, and 10,000 for the time-invariant condition excluding X. In the time-varying conditions, $T = 100$ and X is on in 50 % of the occasions, slow change in X means X is only on during only the first or only the second half of the occasions, fast change means X alternates between occasions, random change means no temporal structure in X. Under the time-invariant condition, we fitted a time-invariant VAR model to stationary data generated from this model. If generating parameter values are zero, absolute instead of relative biases are displayed.

3.3.3 Comparison to a regime-switching model

We now turn to the comparison between our and Hamaker and Grasman's model (2012). The properties of the Markov process underlying the regime switching in the latter model imply a long-run frequency of the moderator being on in around 60 % of the occasions, as verified by simulation. In Table 3.4, we therefore present results for data generated according to this moderator frequency condition without a temporal pattern in the moderator. The general conditions of this sub-simulation are thus not qualitatively different from the conditions of the earlier simulation, we just look at a specific set of parameter values, and a specific moderator frequency. In their 2012 paper, the authors do not report bias, but coverage rates that are substantially lower for $T = 100$ and get comparable with our results only at $T = 500$ (Hamaker & Grasman, 2012, p. 412), which suggests that the regime-switching model is more demanding in terms of required data. This is not surprising, given that the regime switching model is more flexible in that it allows estimating switching between dynamic regimes from the data, whereas fmTSA incorporates switching times as known. Note that the regime-switching model is thus the more appropriate approach, if sources of IA heterogeneity are unknown. However, if information about sources of heterogeneity are available, this comparison shows that they should (and how they could) be used with shorter TS.

3.3.4 Temporal patterns in the moderator

To explore whether temporal patterns in moderator occurrence have an effect on model performance, we compare two extreme temporal structures. A very slowly changing moderator was created by switching the moderator after the first half of the occasions, so that it is on during the first half, and off during the second half, or vice versa. A very fast changing moderator was created by switching the moderator at every occasion. The overall frequency of the moderator remains 0.5 for both the slow and the fast condition. Additionally, we modified the generating model from Equation (3.10) such that there is no difference between the two conditional processes (i.e., the values superscripted by $(X = 0)$ also hold if the moderator is on). The motivation for doing this is a similar to the one that let us simulate data from a symmetric **B** matrix, namely reducing complexity. Specifically, having conditional effects of identical size avoids differences in bias due to differences in size of true effects between the conditional processes. Note that this does not pose any identification problems for the model, as the amount of data available to estimate each conditional process remains the same. We should simply recover that the moderator has no effect and that there are no parameter differences between the two processes. A comparison of how the model in switch-parameterization performs under the different temporal conditions, including the earlier setup in which the moderator changes are just random over time (i.e., the condition "Change X is random"), is included in the upper part of Table 3.5.

Interestingly, the biases in the AR effects (and hence also the compensatory biases in the intercepts) are *largest* when the moderator changes slowly, and *smallest* when the moderator changes fast. Remember that AR (and CR) effect estimates are subject to finite sample bias in general, hence, also in time-invariant models (Cheang & Reinsel, 2000; Marriott & Pope, 1954). The lower part of Table 3.5 therefore shows biases in a *time-invariant* VAR model fitted to a TS of length $T = 50$. We used the parameter values superscripted by $(X = 0)$ from Equation (3.10) to generate the time-invariant data. In comparing the upper and the lower parts of Table 3.5, it seems that the biases occurring in the time-invariant TSA solution to a TS of length 50 represents the limiting case for the biases occurring in a fmTSA solution to a TS of unconditional length 100 and conditional length 50.

We conjecture that the beneficial effect in the fast condition arises in the following way. As mentioned earlier, conditioning on the moderator in fmTSA implies looking at TS that are subsets of the original TS and covary in length with moderator frequency. However, the situation is not exactly the same as looking at these TS subsets independently. Due to the recursive nature of the Kalman Filter, we always make use of the *entire* TS, even when estimating the model parameters conditional on a moderator. Moreover, we make use of the entire series *optimally*, when the two conditional TS are most intermixed. The conditional TS are most intermixed, if the moderator changes quickly, that is, in the extreme, it switches between being on and being off continuously. The conditional TS are least intermixed, if the moderator changes slowly. The estimation of each conditional parameter set relies on the process-predictions available from the Kalman filter for the entire TS. In the intermixed case, any process state prediction generated under one moderator value is still highly informed by earlier process predictions generated under that very same moderator value. This may then improve estimation accuracy for the AR (and CR) effects.

3.4 *Application to data on daily affective experiences from the COGITO study*

3.4.1 Study design, participants, and measures of the COGITO study

The COGITO study is a comprehensive longitudinal study conducted at the Max Planck Institute for Human Development, Berlin, during the years 2006 to 2007 and 2009 to 2010 (Schmiedek et al., 2010). The heart of the study is an intensive longitudinal phase, which tracks cognitive and affective development of 101 younger and 103 older adults over 101 measurement occasions and 158 days, on average. To be measured, participants came to the laboratory every 1.5 days on average, so measurement occasions are irregularly spaced within persons. The daily laboratory sessions were mainly devoted to a broad range of cognitive tasks, but participants also reported affective experiences, self-regulation, motivation, stress experiences, and daily events.

The present analyses are based on the younger participants ($N = 101;\ 51.5$ % women; age: $20 - 31, M = 25.6$; daily sessions: $87 - 109, M = 101$; days in study: $116 - 372, M = 165$) and their reports on negative affect as assessed by the Positive and Negative Affect Schedule (Watson, Clark, & Tellegen, 1988), perceived stress as assessed by the Perceived Stress Scale (Cohen, Kamarck, & Mermelstein, 1983) and daily events assessed within seven different event domains (e.g., work, health, leisure, finances). For each domain, participants were asked to report whether they had experienced an event since the last session, whether an event was ongoing or whether they were expecting to experience an event later that day (e.g., "Has something happened at work or during professional training or studying?"; cf. Brose et al., 2013). They were then asked to rate the event in terms of valence (from "negative", to "neutral", to "positive") and personal relevance (from "doesn't move me" to "moves me a lot").

3.4.2 Model building

As an aspect of daily affective life, we model the joint dynamics of negative affect and perceived stress. For negative affect, we use the average score across the five most variable items (i.e., "distressed", "upset", "irritable", "nervous", and "jittery"), which were answered on an 8-point scale. For perceived stress, we use the average score across two items related to perceived control (e.g., "To what extent do you today feel able to control the things in your life?"), also answered on an 8-point scale. Note that the scale is reversed, so lower values mean higher perceived stress. By applying the time-invariant bivariate VAR model (Equation (3.1);

Model 1) we assume that an individual's current affect (corresponding to η_1) and stress levels (corresponding to η_2) depend on earlier affect and stress levels, but also on new, independent input at that particular occasion, such as situational influences or internal events unrelated to earlier affect or stress (i.e., the residuals). As before, we fix the measurement model so that there is a one-to-one relationship between observed and latent variables.

Extending the time-invariant baseline model, we allow all model parameters to vary intra-individually, conditional on daily events. We confine the analysis to events that were reported to have happened before the session or were still ongoing. We then collapse across event domains and look at whether events were reported as negative and as being somewhat relevant (at least "moves me a little"). The information on these negative events is incorporated as a dichotomous and a continuous-valued moderator.

For the dichotomous moderator case (Model 2), we set up a dummy coded variable which indicates whether at least one relevant negative event occurred before a given occasion on the same day. Included as a moderator (cf. Equations (3.5) and (3.6)), this implies that parameters may be different on occasions preceded by relevant negative events as compared to occasions not preceded by those events (i.e., occasions preceded by positive, neutral, or irrelevant negative events). We set up the model according to the change-parameterization.

For the continuous-valued moderator case (Model 3), we calculate a linear weighted moving average of relevant negative events as an index of "stressor pile-up" (Schilling & Diehl, 2014, p. 72 ff.) as

$$Z_t = \sum_{k=1}^{K} \left(X_{t-(k-1)} \left(K - (k-1) \right) \right) \left(\sum_{k=1}^{K} \left(K - (k-1) \right) \right)^{-1}, \tag{3.12}$$

where Z_t is the continuous-valued moderator, X_t is the dichotomous moderator and K is the size of the window over which we average. This index reflects the accumulation of relevant negative events over K consecutive occasions up to and including the current one. In being accumulated, events get less weight the earlier they occurred. Distance from the current occasion is thereby taken into account linearly. In the present analyses, we set $K = 3$, so that $Z_t = (3X_t + 2X_{t-1} + X_{t-2})(3 + 2 + 1)^{-1}$. In the moderated model (cf. Equations (3.5) and (3.6)), Z_{t-1} then replaces X_{t-1}. If Z_{t-1} has a moderating effect, and Model 3 fits the data better than Model 2, it implies that relevant negative events have a stronger moderating impact if they follow up on each other (because Z_{t-1} then takes on higher values) as compared to when they occur in isolation. The first two occasions of this moderator are imputed by the mean. As the continuous-valued moderator was built from the dummy coded moderator, it could not assume negative values and varied in the relatively restricted range between zero and one. We could therefore use simple linear functions to link it to the model parameters without the risk of pushing the parameter estimates pertaining to each conditional process to extreme values.

3.4.3 Model fitting

As described above, measurement intervals are irregularly spaced within individuals in this data set. Since participants could come to the lab on a daily basis, we can think of each day a person did not come in as a missing occasion. In discrete-time TSA, this can be handled readily by the Kalman filter, which generates model-implied predictions of the processes' state also for missing occasions (done in case of Model 1; Durbin & Koopman, 2012). In time-discrete fmTSA, as employed here, the same strategy can be applied to missing values on the process variables. The moderator, however, enters the model as fixed. Hence, the model offers no account of how the moderator behaves over time and therefore has no implications for plausible values for occasions on which the moderator was not observed.

One way to deal with this problem would be multiple imputation (Buuren, 2012; Schafer & Graham, 2002). Here we use a simple imputation method for the moderator: On missing occasions, we sample from the binomial distribution with the empirical probability of success set to the frequency of the dichotomous moderator observable in the available data (i.e., assuming stationarity of the moderator and no temporal structure). We generate 100 imputed data sets per person for the dichotomous moderator case and calculate the continuous-valued moderator based on each of these. Using these imputed and recalculated data sets, we repeatedly fit Models 2 and 3. To a-priori restrict the potential variability between modeling solutions within individuals, we confine this illustration to a subset of nine younger participants with no more than around 25 % missing occasions[3]. An additional criterion guiding the selection of cases to analyze was variability in the moderator, leading us to leave out participants with relative event frequencies below .18, and above .82. Appendix C shows the trajectories of the selected cases for the sum scores of negative affect and perceived stress with

[3] Intra-class correlations (i.e., the amount of between-imputation variance over the amount of total variance) are <.10 for almost all parameters across individuals for Model 2 and <.14 for all parameters and cases except Cases 3 and 9 for Model 3. This indicates that the amount of variation between the 100 solutions based on the imputed data sets is relatively small.

the dichotomous moderator visualized in the background. Per individual, we pool solutions across imputed data sets (i.e., point estimates and standard errors, calculation of within-imputation, between-imputation, and total variance) according to Rubin's rules as described in Schafer & Graham (2002).

We initialize the Kalman filter via the lag-zero moments of the observed TS. In fitting the models, we make use of (a modified version of) the OpenMx function *mxTryHard* that promotes and monitors iterative refitting and reuses previous parameter estimates as new starting values.

3.4.4 Model comparisons

Within each person, we thus fit three different models, the time-invariant baseline model, Model 1, the dichotomously moderated Model 2, and the continuously moderated Model 3. For comparisons of Models 1 versus 2, and Models 1 versus 3, we rely on likelihood ratio testing ($\alpha = 0.05$) since the models are nested. We pool χ^2-values as described in Buuren (2012). Additionally, we report the Akaike information criterion (AIC; Akaike, 1974) as averaged across solutions, whereby a lower average AIC implies better fit. We do not include both moderators simultaneously and can therefore not statistically test for an incremental effect of the continuous-valued over the dichotomous moderator (note that the two are correlated since one is built from the other). Instead, we rely on the AIC and the actual modeling solutions.

Table 3.6. Model comparisons across individuals

Case	Model 1	Model 2		Model 3	
	AIC	AIC	χ^2 (vs. Model 1)	AIC	χ^2 (vs. Model 1)
1	**268.17**	272.47	13.70	282.62	3.56
2	456.40	**375.68**	**98.72***	**393.01**	**81.39***
3	269.47	**257.53**	**29.95***	**266.21**	21.26
4	551.46	**551.32**	18.14	553.81	15.65
5	407.22	**388.60**	**36.63***	**395.81**	**29.41***
6	556.20	**554.02**	20.18	-	-
7	**342.73**	348.11	12.62	347.15	13.58
8	570.08	**526.24**	**61.84***	**549.95**	**38.13***
9	449.33	**444.92**	22.41	**449.20**	18.13

Note. * p < (.05/m), m=9 for Model 2 and m=8 for Model 3. χ^2-values are pooled, the test statistic for the pooled χ^2-values is F-distributed under the null-hypothesis of equal model fit with degrees of freedom as reported in Buuren (Buuren, 2012, p. 159). The best fitting model per comparison per case is printed in bold, the best fitting model per case is printed in bold and underlined.

3.4.5 Results

Figure 3.3 and Figure 3.4 display the solutions of Models 2 and 3 in terms of 95% CI estimates across individuals. Table 3.6 summarizes the model comparisons.

We first consider Model 2. By incorporating the dichotomous moderator and thus discretely time-varying parameters into the time-invariant baseline model (Model 1) global model fit improves for seven individuals according to the AIC, and for four individuals according to likelihood ratio testing (Bonferroni corrected). Looking at the averaged profile likelihood-based CIs in Figure 3.3, and here especially at the parameters that estimate change (Panel B of Figure 3.3), we see different effects for those individuals for whom there is a decrease in AIC from Model 1 to Model 2 (i.e., Cases 2 to 6, 8, and 9 as highlighted in Table 3.6). The residual variances of negative affect seem to be moderated most consistently, that is, for four individuals (i.e., "px11" in Panel B of Figure 3.3). These are also the cases for whom the inclusion of the moderator leads to a statistically significant increase in model fit (i.e., Cases 2, 3, 5, and 8 as highlighted in Table 3.6). Case 8 additionally has a negative main effect of the moderator on perceived control (i.e., "ax2" in Panel B of Figure 3.3). The moderated residual variances indicate increased variability in negative affect following negative events, after taking into account lagged effects of earlier affect and stress.

We now turn to Model 3 and the continuous-valued moderator. Revisiting Table 3.6, we see that, in comparison to Model 1, global model fit is improved for three individuals according to likelihood ratio testing (Bonferroni corrected). As for Model 2, those cases (now Cases 2, 5, and 8) are the ones showing moderated residual variances of negative affect (i.e., "px11" in Panel B of Figure 3.4). Case 8 additionally has a negative main effect on perceived control (i.e., "ax2" in Panel B of Figure 3.4), as under Model 1. As shown in Table 3.6, there is an AIC decrease as compared to Model 1 for five cases, however, the AIC favors Model 2 for all individuals. Note that there were some problems when fitting Model 3. That is, we did not get any converging solutions for Case 6 (i.e., NPSOL status 6) who is therefore not included here. Also, across the 100 imputed data sets, 75 modeling solutions did not converge for Case 1, and 11 solutions were non-convergent for Case 3, among them five conditionally unstable solutions (i.e., at least one eigenvalue of $\mathbf{B}^{*}$ and/or $\mathbf{B}^{(0)}$ was equal to or larger than one in absolute value).[4]

[4] These problematic instances seem to suffer from problems of empirical identification. That is, Cases 1 and 6 have the lowest relative moderator frequencies in the subsample selected (both around .18). Additionally, occurrences of the dichotomous moderator are unequally distributed across occasions, leading to little redundancy in values of the continuous moderator. Case 3 displays very little variation in perceived stress over time.

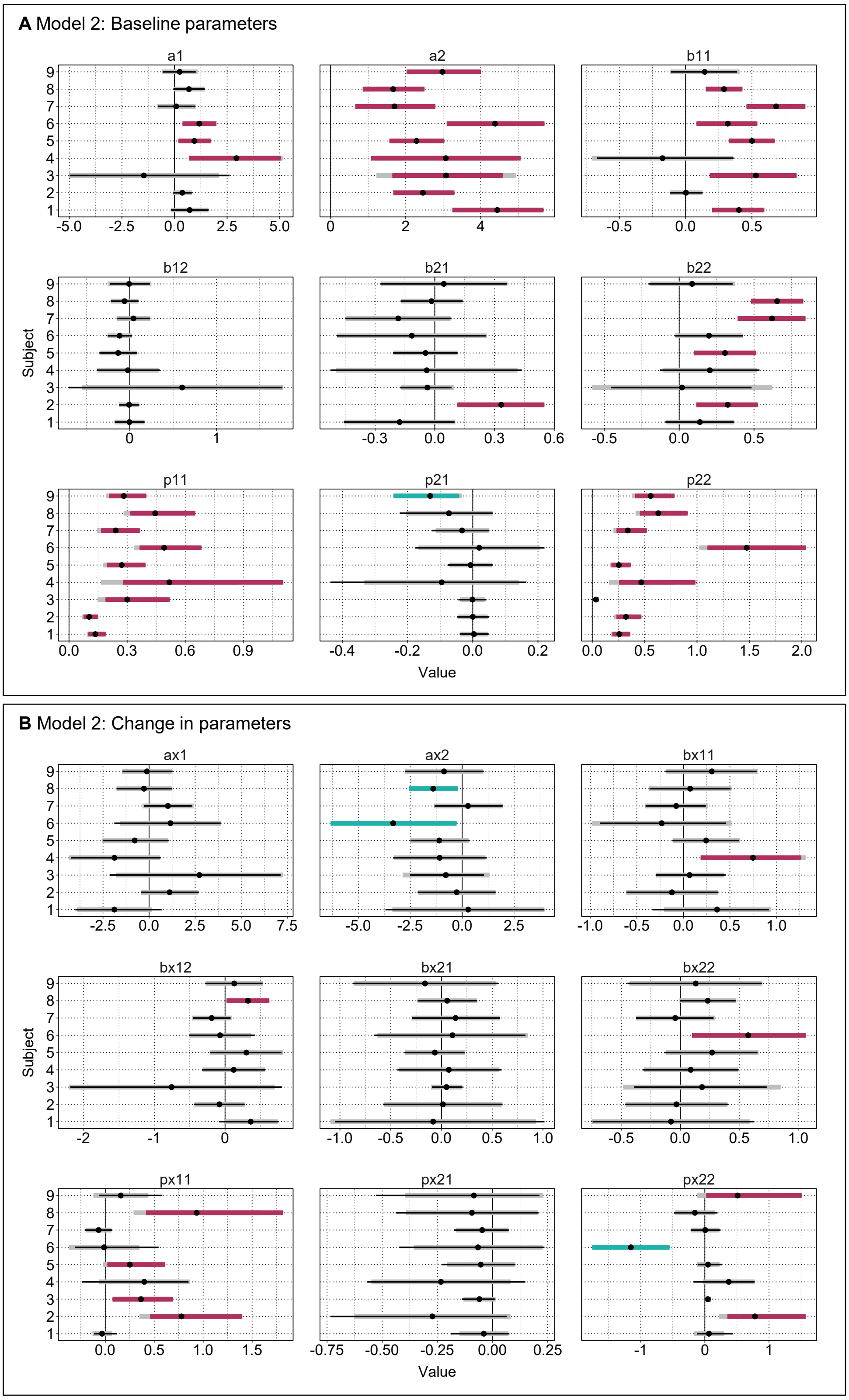


Figure 3.3. Individual solutions under Model 2 in terms of point and 95% CI estimates per parameter. Baseline parameters are displayed in Panel A, change in parameters is displayed in Panel B. Profile likelihood-based CIs, averaged across solutions, are shown in black if including zero, in green if excluding zero and being negative, and in red if excluding zero and being positive. Wald-type CIs based on the pooled standard errors are shown in grey. The pooled point estimates are displayed as points.

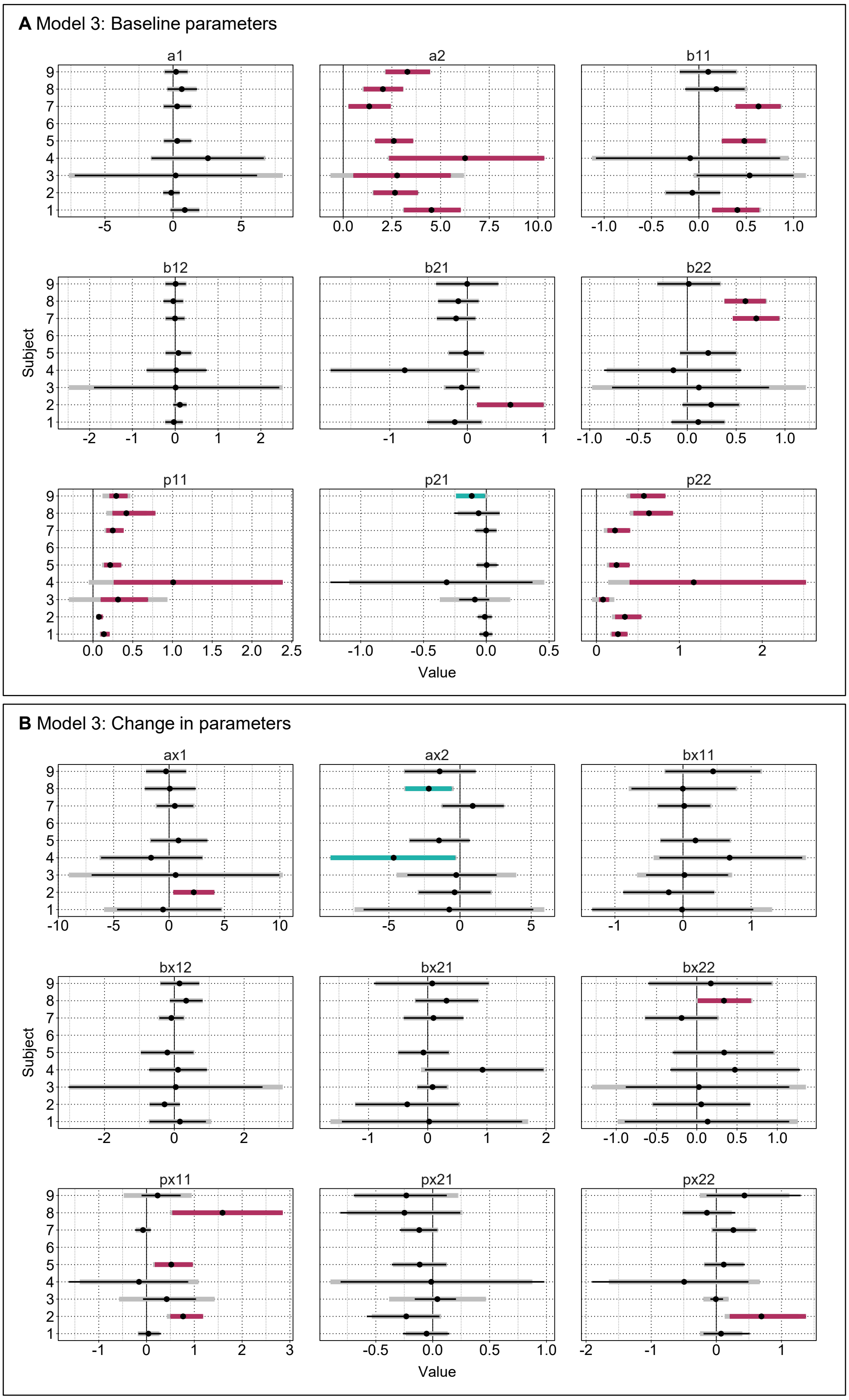


Figure 3.4. Individual solutions under Model 3 in terms of point and 95% CI estimates per parameter. Baseline parameters are displayed in Panel A, change in parameters is displayed in Panel B. Profile likelihood-based CIs, averaged across solutions, are shown in black if including zero, in green if excluding zero and being negative, and in red if excluding zero and being positive. Wald-type CIs based on the pooled standard errors are shown in grey. The pooled point estimates are displayed as points.

3.4.6 Discussion

Looking at IA affective functioning within the young sample of the COGITO study, there is some evidence for moderating effects of relevant negative events on the joint dynamics between negative affect and perceived stress. The most prominent effect, present in about half of the subsample, was an increase in the variance of negative affect unexplained by earlier negative affect and perceived stress after a relevant negative event had happened. We found no evidence for negative events having cumulative effects on the model parameters.

The observed effects on the process residual variances can be interpreted in different ways. It might be that some individuals are more (or more often) unstable in their negative affect levels following days with negative events. These effects might be hidden dynamic effects, arising from temporal person-situation interactions that occur fast as compared to the temporal resolution of the current data and thus do not show up as lagged effects. Alternatively, the model might be reflecting a characteristic of the situation rather than the person, in that the days following negative events could have been more eventful. We can examine this post hoc to the extent events have been assessed. Here, the zero-order IA relationships between daily events of positive and negative valence, for instance, range from negative, to zero, to positive in the COGITO data. For a given individual, we may thus be contrasting days with negative events to days with positive events, days with both types of events to days without those events, or days with negative events to days with or without positive events. However, looking at the co-occurrence of relevant negative and positive events within the selected cases reveals that relationships are close to zero or negative (Spearman's rho ranges between .06 and -.22).

Some caution is advised, as individuals contribute conditional TS (given the moderator) of different lengths and thus of different degrees of bias and coverage rates, especially relevant for estimating (change in) the residual variances. Sorting solutions according to event frequency does however *not* reveal a clear trend (i.e., underestimation of change in residual variances for low moderator frequencies, overestimation for very high frequencies under the change-parameterization; from low to high event frequencies, the order of cases is 1, 6, 9, 2, 5, 8, 7, 3, 4). Also, in this application, we may well face a situation in which statistical power is sufficient to detect larger effects only. For Model 3, the imputation method is assumed to have added some noise too, as there was no temporal structure in the imputed dichotomous moderator data which were used to calculate the continuous-valued moderator. So, the absence of evidence for moderating effects is not necessarily to be taken as evidence for their non-existence.

Since we use one indicator per construct, we stick to a constrained version of the measurement model in which we fix the factor loadings to one and the measurement residual variances to zero. With one indicator, the distinction of process and measurement residuals becomes relatively unstable unless the AR effect is very high or the TS are very long. Multiple indicator models are an alternative, which we not consider in this illustration.

3.5 Discussion of potentials and limitations

In this chapter, we propose fmTSA to study non-stationarities in affective processes due to time-varying process parameters. Such non-stationarities may for instance arise as a function of a person's interactions with specific situations in daily life. In the following discussion, we turn to potentials and limitations of the proposed modeling approach. This presentation is structured according to two main qualities of the model, namely, it being a model for observed heterogeneity and it being a TS model applicable to data from single individuals.

3.5.1 Potentials

The model presented here is one for observed IA heterogeneity as it treats the source of heterogeneity as known (i.e., the model features parameters that vary as a *deterministic* function of a *fixed* time-varying moderator). The model thus makes assumptions about the shape or pattern of temporal variability or change in model parameters while freely estimating the amount of change.

An advantage of this approach to IA heterogeneity is its practicality. This concerns model implementation and estimation, in that incorporating IA heterogeneity as observed yields a standard optimization problem for which solutions are widely implemented (e.g., the Kalman filter implementation in OpenMx; Hunter, 2014b). It also concerns feasibility in relatively small samples. As shown in the simulation, the model can be expected to be applicable given small to medium numbers of measurement occasions (e.g., $T = 100 - 200$) if there is sufficient variability in the moderator. With small sample sizes, however, problems of finite sample bias occur. The use of analytical or bootstrapped bias approximations for correction is possible (cf. Cheang & Reinsel, 2000). Here, we rely on profile likelihood-based CI estimates that show acceptable coverage rates even with small T, as long as there is sufficient variability in the moderator. An interesting finding from the simulation study concerns the effects of temporal regularity in the moderator on model performance. In particular, it seems that more alternations between conditional processes are associated with less bias and higher coverage rates in the AR effects. We conjecture that in case of a more quickly changing moderator, parameter estimation via the Kalman filter makes better use of the entire TS data available.

Conceptually, the model incorporates and tests specific hypotheses about change in parameters. Despite its reliance on a priori knowledge about the potential sources of IA heterogeneity, the model retains quite some flexibility. That is, time-varying moderators of various formats can be readily incorporated and thus parameter changes of very different shape can be tested within a single modeling framework. We consider this another advantage of the presented approach. In the application to the COGITO data, we investigated discrete-valued parameter changes as a function of negative events being present or absent as well as more continuous changes related to the accumulation of negative events over time. In an illustration of the model's behavior using simulated data, we also demonstrated that one is not restricted to substantive moderators or fluctuating parameter change. It is also possible to consider (e.g., gradual) changes as a function of time. Another flexibility regarding shapes of change concerns the functional relationship between the moderator and the model's parameters, which can be freely specified as long as it is deterministic. Also, hypotheses about change can be incorporated in a parameter-specific manner. Finally, models for unobserved heterogeneity have been criticized to run the risk of misrepresenting non-normality not related to genuine heterogeneity as "spurious classes" (Bauer, 2007, p. 782). This is less likely to happen with a hypothesis-driven account to heterogeneity.

A distinguishing characteristic and important potential of the model is that it is a TS model applicable to data from *single* individuals. We argued that fmTSA therefore provides the capacity to unconstrainedly examine the complex and profound IE differences that IA psychological phenomena are potentially subject to. Such a bottom-up approach seems appropriate in cases of extreme IE differences, when aggregations across individuals must remain on a descriptive, informal level by virtue of the phenomenon under study. This is of course an extreme case and it has been argued that heterogeneity should be considered a gradual feature rather than an all-or-none situation (Brose, Voelkle, Lövdén, Lindenberger, & Schmiedek, 2014; Voelkle, Brose, Schmiedek, & Lindenberger, 2014). It may well be that individuals indeed share certain characteristics, hence, comprise a somewhat unified population. Based on this, one might then formulate an average IA model structure and a formal between-subject model, which captures individual deviations from the average structure. The moderated VAR model presented here may be a candidate for an average within-subject model. If feasible, an integrated model affords simultaneous statistical inference intra- as well as interindividually and can increase the precision of IA estimates (von Oertzen & Boker, 2010). Note that, in these situations, single-subject modeling can still be a valid first step if a phenomenon is not well known and an explorative account is sought. Also, it can support model building (i.e., setting up appropriate priors in (Bayesian) hierarchical modeling; Schuurman, Grasman, & Hamaker, 2016).

3.5.2 Limitations

Obviously, the fact that heterogeneity needs to be observed may be considered an advantage of the approach – but also a disadvantage. If no potential moderators are assessed or if hypothesis about their impact or about patterns of parameter change over time are lacking, the model is not applicable. That said, one need not have very strong hypotheses about parameter change in order to apply the model. A more explorative use seems warranted if backed up by subsequent effort to replicate and validate the captured instantiations of IA heterogeneity and thus ensure their psychological substance. Of course, incorporating observed sources of heterogeneity does not preclude to possibility of unobserved heterogeneity in the data remaining unaccounted for.

Another limitation is that in the suggested fixed moderator approach, the moderator itself is not modelled. Hence, the model offers no account of how the moderator changes over time. If the moderator is a substantive covariate, this can pose the following problems. First, missing values on the moderator cannot be handled readily *within* the model. This also concerns attempts to take unequal measurement intervals into account by incorporating missing values or by taking a time-continuous perspective (Driver, Oud, & Voelkle, 2017; Voelkle, Oud, Davidov, & Schmidt, 2012). Fortunately, as these are problems shared with a broad class of other modeling approaches that include fixed or exogenous variables (e.g., standard regression and multi-level modeling, VAR models with covariates), strategies to cope with this have been developed. Multiple imputation (Buuren, 2012; Schafer & Graham, 2002) is an acknowledged solution, for instance. Note that treating the moderator as fixed is also problematic if observed moderator scores are rather noisy, error-prone reflections of the *true* moderator scores. Distinguishing between these two sources of variation, noise and signal, is possible in the context of a measurement model. Including a measurement model for the moderator, however, requires shifting to models that incorporate interactions (i.e., non-additive effects) among latent variables (e.g., Klein & Moosbrugger, 2000).

Finally, empirical identification can be an issue, as we have seen in the illustration. Since the model decomposes (co-)variances, parameter estimation is complicated in the presence of limited variance in both the process variables and the moderator.

3.6 A future application: exploring affective flexibility in a virtual reality

A recurring theme in the micro-longitudinal affective functioning literature is the search for *adaptive* patterns of affective functioning (see Chapter 2, Section 2.3.3). Whether affective dynamics are adaptive, is usually decided upon their (supposed) capacity to maintain or increase positive emotional outcomes (e.g., well-being) or prevent negative ones (e.g., depression) on the long run. This perspective on affective functioning is of interest, whenever psychological health or "successful" development (cf. Rowe & Kahn, 1997) becomes a target of optimization out of individual or socioeconomic reasons.

In structuring ideas about affective adaptations to the various perturbations and challenges of our daily lives, the dimension of *flexibility* seems to be an important one (Kashdan & Rottenberg, 2010). Some of the currently popular affective conceptions can be aligned along this dimension (Aldao et al., 2015; Bonanno & Burton, 2013; Kuppens, Allen, et al., 2010; Malooly, Genet, & Siemer, 2013; Rottenberg, Gross, & Gotlib, 2005). A framework for affective flexibility at different time scales that is motivated by the micro-longitudinal paradigm – the "Flex3" model (Hollenstein et al., 2013) – has recently been proposed.

Here, I suggest using the "Flex3" model as a heuristic framework to explore *flexibility in affective dynamics* across contextual variations using fmTSA and data from an experimentally controlled virtual environment.

3.6.1 Affective flexibility in affective dynamics across contexts

The "Flex3" model is a rather descriptive model that distinguishes affective flexibility within specific contexts ("dynamic flexibility") from flexibility across contexts ("reactive flexibility") and more enduring changes in dynamic and reactive flexibility ("trait or developmental flexibility") due to development or major life events (Hollenstein et al., 2013, p. 1 ff.). With the focus on "reactive flexibility" at the intermediate layer of the model, we can contrast flexibility within contexts with flexibility between contexts. Considering this minimal hierarchical version of the model allows examining flexibility in a *systemic* way, that is, it allows investigating how the temporal structure of an affective system reacts to contextual changes. Substantively, this is interesting, as this may reveal contextual effects on how affective processes evolve over time, for instance due to state-dependent emotion regulation (Adolf et al., 2017; De Haan-Rietdijk, Gottman, et al., 2016).

3.6.2 Fixed moderated time series analysis

Statistically, "reactive flexibility" may concern changes not only in (residual) means but also in the variance-covariance structure of affective processes and thus the processes' dynamics over time. It therefore corresponds to the situation of *full* IA heterogeneity and requires an appropriate modeling solution.

In the previous sections, we presented fmTSA, which allows including context as a moderator of the parameters of a dynamic TS model. Technically, it is a model for observed IA heterogeneity in that it accounts for change in model parameters that follows a fixed shape and estimates the extent to which the presumed changes play out. Due to this property, the model is straightforward to implement and estimate.

Given for instance the familiar AR model as a potential prototypical candidate for an affective process model, we can look at different manifestations of "reactive flexibility". First, we might look at main effects of context on affect, that is, context affecting affect levels (e.g., a person might experience more negativity on average during some situations as compared to others). Second, we might look at interaction effects in the sense that carry-over effects depend on context (e.g., it might take a person longer to regulate back or recover from a perturbation during some situations as compared to others). Third, we can look at interaction effects in the sense that the effect of the stochastic process residuals depends on context (e.g., a person may be more sensible to perturbations during some situations as compared to others).

However, treating contextual changes as known and their impact on model parameters as deterministic might be problematic in observational data. As discussed earlier, situations in which there are no clear candidates for moderating contextual factors may only provide for a rather explorative use of the model with the need to replicate findings. Also, if the timing of contextual changes is uncertain due to measurement error or if contextual changes are non-exogenous (e.g., affected by earlier affective states), estimating moderating effects using fmTSA may lead to biased results.

In the following, we present data that to some extent circumvent these difficulties and seem therefore well suited for an application of fmTSA to explore affective flexibility.

3.6.3 Data from a virtual environment

The data in question are experimental data gathered in the context of the virtual environment Room 101, which is part of a set of virtual environments designed to elicit affective experiences (McCall et al., 2016). Room 101 consists of a room with a latently fear-eliciting atmosphere, in which participants then experience “epochs” of different threat levels, featuring sustained ambient changes (e.g., changes in background sound and lightning), as well as the occurrence of discrete events such as explosions, spiders or snakes appearing, and eventually the whole room collapsing. During the course of the experiment, people were supposed to exploit the room, guided by the task to collect objects. Physiological affective experiences (i.e., skin conductance and heart rate) were measured online, subjective affective experiences (i.e., arousal) offline during a replay (for a detailed description see McCall et al., 2016, p. 101 f.). Covariates of potential interest include trait resilience and emotional task switching (L. K. Hildebrandt, McCall, Engen, & Singer, 2016).

A characteristic of the data important for the present purpose is that situational aspects are the target of experimental manipulation and are thus under experimental control – although there remain some degrees of freedom, as people could move somewhat freely through the room. At the same time, the scenario is realistic and convincing, so, Room 101 seems to strike a good balance between internal and external validity. An exploration of the data using fmTSA to estimate systemtic affective reactions to contextual changes that are assumed to be known and exogeneous, seems thus warranted.

3.6.4 Potential analyses and outlook

Various questions have been tackled using the Room 101 data. They include the coherence of the different affect measures over the experiment in relation to IE differences in interoception (McCall, Hildebrandt, Bornemann, & Singer, 2015), movement through the room and gaze behavior during the experiment in relation to IE differences in self-reported fear (McCall et al., 2016), and change in IE associations between affective states and emotional task switching, self-reported resilience, and heart rate over the experiment’s course (L. K. Hildebrandt et al., 2016).

IA affective change and dynamics, and especially change in affective dynamics as a function of the epoch the room is in, has not been looked at so far. It seems however plausible, that the different epochs created to exert changes in threat level, function as emotionally relevant contexts that might evoke more systemic affective reactions and thus changes in within-epoch affective dynamics.

Some of the data analytic difficulties concern the potential inclusion of response functions for the physiological affect signals, the selection of appropriate (parametric) dynamic models and/or the data-driven estimation of and control for slow non-linear trends present in the data, and dealing with sampling rates that are so high, that the data contain periods of non-changing information, which are unlikely to be meaningful.

4 Poisson and Gaussian vector autoregressive modeling of context and affect

Recent theoretical contributions to the affective functioning literature call for more explicit considerations of the contexts people make affective experiences in to better understand daily affective functioning (e.g., Aldao, 2013). Daily events and specifically *stressful daily events* are a contextual feature considered important in eliciting regulatory processes and shaping affect in everyday life (e.g., Bolger & Zuckerman, 1995). Events are also a relatively accessible contextual characteristic, and a number of studies has investigated daily stressors in relation to affective experiences within person (e.g., Brose, Schmiedek, Lövdén, & Lindenberger, 2011; Koval et al., 2015; Suls et al., 1998; Zautra et al., 2002). However, the statistical approaches typically employed (i.e., within the ordinary mixed effects regression framework) fall short on putting changes in events and changes in affect on equal footing. That is, they entail a clear distinction between endogenous and exogenous variables and include the latter as fixed regressors that are themselves not modeled. As a consequence, the possibilities to formalize and test ideas about the interplay between affect and events are limited, and events often feature only as correlates or antecedents of affective states.

In this chapter, I am therefore concerned with setting up a dynamic model for changes in daily events, specifically the number of stressful events, and a joint dynamic model for changes in stressful events and affective experiences. This does not only conceive of environmental changes as instantiations of systematic processes, and targets potentially interesting temporal dynamics in stressor occurrence; the simultaneous modeling of changes in events and affect as bi-directionally interacting processes also enables addressing more sophisticated substantive questions about the interplay of contextual and affective processes – and provides a more comprehensive picture of daily affective functioning.

To take into account the characteristic distribution of event counts, I adopt a Poisson autoregressive (PAR) model (Brandt & Sandler, 2012), which has been developed outside psychology to analyze multivariate event count time series (TS). I modify the model so that it adheres to the autoregressive (AR) model structures typically used in psychological applications (Browne & Nesselroade, 2005; Hamaker & Dolan, 2009) and so that it accounts for event dynamics in multiple individuals simultaneously. In the modified setup, the model involves a latent process model and a measurement model, and thus addresses the dynamics of daily events at the level of continuous-valued latent variables, in this case event rates (Brandt, Williams, Fordham, & Pollins, 2000). I then incorporate the univariate PAR model into a bivariate process model, along with an AR model for fluctuations in affective experiences. Model estimation rests on stochastic simulation using Markov chain Monte Carlo (MCMC) techniques and is carried out in a Bayesian framework.

The outline of this chapter is as follows. I first set out a process perspective on daily stressors. This involves introducing the concept itself, as well as how daily stressors may matter *in quantity* for affective functioning. I then argue that conceiving of changes in daily stressor counts as subject to time-structured dynamics is an informative but rarely taken perspective. To formally implement such a perspective, the *Poisson* distribution may be incorporated into existing dynamic modeling approaches. I thus briefly lay out the mathematical properties of this distribution function. Second, I present the linear PAR model proposed by Brandt & Sandler (2012), my modified version of the model, and the bivariate extension of the model for the joint dynamics of stressors and affective experiences, a hybrid Poisson-Gaussian vector autoregressive (hPVAR) model. Third, model estimation as carried out in the Bayesian framework is explained. This is followed by, fourth, a simulation study that explores the performance of the proposed models. Specifically, I investigate accuracy and efficiency of Bayesian point and interval estimates. I fifth show an application to data from the COGITO study (Schmiedek, Bauer, Lövdén, Brose, & Lindenberger, 2010), relying on self-report data of stressful events from different life domains and negative affect. I close with a discussion of potentials and limitations of the proposed models with respect to both technical and substantive aspects.

4.1 An (extended) process perspective on daily stressors

4.1.1 Daily stressors matter in quantity

Emotions may be conceptualized as quick and ongoing reactions to changing *environmental demands* that facilitate adaptive behavior (Aldao, 2013; Gross, 1998b; Kuppens, Allen, et al., 2010). Specific environmental demands often studied are stressful events in everyday life, also termed daily stressors or daily hassles (e.g., Lazarus & Cohen, 1977). I use these terms in an exchangeable manner in the following. Kanner and colleagues (1981, p. 3) describe daily stressors as “the irritating, frustrating, distressing demands that to some degree characterize everyday transactions with the environment” (see also DeLongis, Coyne, Dakof, Folkman, &

Lazarus, 1982; Wagner, Compas, & Howell, 1988). There are also suggestions as to *how* daily stressors might function as a source of stress and consequently affect affective experiences: In contrast to major life events that occur rather rarely and thus stand on their own, daily hassles are assumed to be rather “stable, repetitive, or chronic” (Lazarus & Cohen, 1977, p. 93) and should thus take effect in a cumulative manner, by “piling-up over a series of days to create persistent irritations, frustrations, and overloads” (Almeida, 2005, p. 64; see also DeLongis et al., 1982; Kanner et al., 1981; Schilling & Diehl, 2014). Hence, the *number* of hassles encountered in a critical time period may qualify as a quantitative, gradual marker of stressful environmental demands and may be considered one of the “objective characteristics of daily stressors” (Almeida, 2005, p. 65).

4.1.2 Process perspectives on daily stressors

A number of studies has targeted such objective characteristics of daily hassles within person over time, in terms of the number of stressful daily events or in terms of stressful events being present or absent (Bolger & Schilling, 1991; Brose et al., 2011; Kanner et al., 1981; Koval et al., 2015; Sliwinski, Smyth, Hofer, & Stawski, 2006; Suls et al., 1998; Zautra et al., 2002). Typically investigated are intra-individual (IA) relationships between affective experiences and stressful events, either as undirected, contemporaneous associations or as directed effects of earlier events on later affect. Fluctuations in the occurrence of daily stressors over time, however, are usually not modelled explicitly. As a consequence, potential effects of earlier affect on later events are also not accounted for. Judging from the typically implemented modeling approaches, it thus seems as if the notion of stress as a process (Almeida, 2005, p. 66; Bolger & Zuckerman, 1995) concentrates on the processes that “mediate between the environmental stressor and the stress response” (Lazarus & Cohen, 1977, p. 108) and thus the processes that unfold within the individual in relation to appraisal of or reactivity to a stressor.

But changes in daily environments are likely to be non-random and thus also subject to processes with specific temporal dynamics. The dynamics that structure daily stressor occurrence are an interesting aspect of a person’s environment in their own right. One may for example ask whether individuals are exposed to an environment with only slow changes in the amount daily hassles or whether the number of hassles is fluctuating rather quickly and unstructured. Whereas an environment displaying slow changes might be more predictable for an individual (cf. Brose et al., 2013), but may also come with prolonged periods of high stress (i.e., higher numbers of stressors), a faster, less structured pattern implies less predictability, but may also offer more possibilities for an individual to change his or her environment.

If one now, based on this, conceives of intra-personal and contextual processes as coupled processes, it becomes possible to quantify the extent to which fluctuations in stressors (statistically) structure later fluctuations in affective experiences, and the extent to which fluctuations in affect structure later fluctuations in stressors. For both contextual and intra-personal changes a process perspective thus allows drawing the distinction between temporal patterns potentially induced by the respective other side, and temporal patterns existing independent of the respective other side. While effects of earlier events on later affect may indicate processes of affective reactivity, effects of earlier affect on later events may indicate processes of stress anticipation or evocation, or reactivity of the environment if other actors are involved.

4.1.3 Modeling change in the frequency of daily stressors via the Poisson distribution

Although not often tested empirically (Voelkle, Ebner, Lindenberger, & Riediger, 2013), the idea that individuals are not only affected by, but also affect (or predict) emotionally relevant parts of their environment is of course contained in the theoretical literature (e.g., Gross, 1998a). It thus becomes even more important to put events and affective experiences on equal footing statistically. To do this, simultaneous dynamic modeling of changes in daily events and changes in affect seems well-suited (i.e., no a priori assumptions about exogeneity or directionality of effects are built into the model). My present account builds on the linear AR model, as introduced in Chapter 2, Section 2.2, as a popular and potentially useful model for intensive longitudinal event and affect data.

In psychological applications, the AR model features continuous-valued and normally distributed variables. The symmetric, bell-shaped normal distribution (also Gaussian in the following), may often provide a reasonable approximation to how psychological measures are distributed or may be a parsimonious candidate if little is known about the distributional shape of a continuous-valued variable in the population (Jaynes & Bretthorst, 2003). The number of daily stressors in a certain time period, however, is not only a non-negative, discrete-valued entity, it may also realize according to very different distributional shapes over time. When stressors are rare and only a few stressors accumulate, one might observe a highly right-skewed distribution. In contrast, there may be patterns, where stressors are more

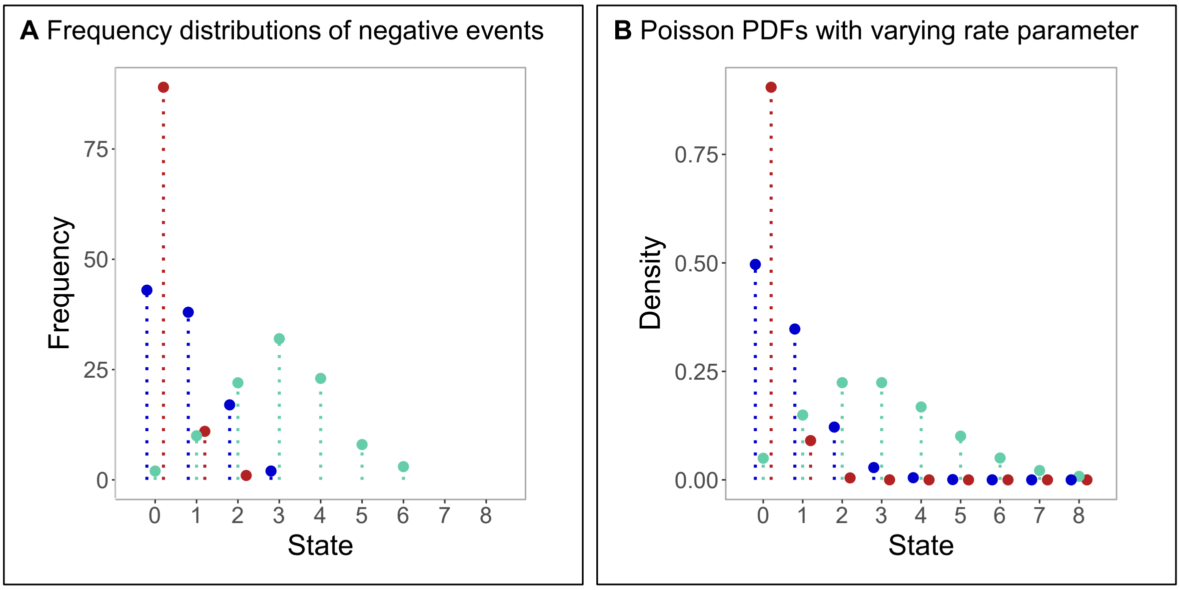


Figure 4.1. Frequency distributions of negative events in comparison to Poisson distributions for different rate parameters. Panel A shows the frequencies with which three COGITO participants, coded by color, report different numbers of occasion-specific negative events, aligned as different states on the x-axis, over the entire study period. The participants are selected such that their reports adhere to different distributional shapes, from highly skewed in red to rather symmetric in green. Panel B shows three Poisson probability density functions for different event rate parameters. It is directly apparent, that the selected event rates lead to a close match in shape between the Poisson probability density functions and the empirical frequency distributions. One again observes shapes from highly skewed in red with a rate parameter of 0.1 to rather symmetric in green with a rate parameter of 3. The blue distribution is generated with a rate parameter of 0.7.

frequent, approaching more symmetric distributional shapes. Figure 4.1 shows such different frequency distributions of negative events as reported by three COGITO participants over the entire study period (Panel A).

Figure 4.1 also shows how these different observed distributional shapes can be matched by the Poisson distribution (Panel B). The Poisson distribution is a standard choice when it comes to modeling the number of events. Of interest is the number of times a certain event occurs in a given time period. Within this time period, one assumes a constant average rate of event occurrence and independence between events. Additionally, longer pauses between successive events are assumed to be less likely than shorter pauses. Specifically, one assumes a negative exponential distribution of pauses, but there needs not be a very high concentration of mass close to zero. As demonstrated in Figure 4.1, a Poisson distributed random variable can accommodate probability distributions of various shapes, from highly right-skewed in the case of low events rates towards bell-shaped, symmetric distributions in case of higher event rates.

So, if I consider negative daily events across distinct life domains within a time period that is relatively short (e.g., within days), it might well be, that the number of these negative events realizes according to a Poisson distribution, that is, independent across life domains, and potentially right-skewed as people might try to avoid or prevent the exposure to daily stressors (Charles, Piazza, Mogle, Sliwinski, & Almeida, 2013; paralleling findings on skewed negative affect, e.g., Schilling & Diehl, 2014).

In line with my earlier reasoning, I then suggest to focus on how the number of negative events expected within time periods (i.e., changes in the event rate) changes *over* time periods (e.g., over days), and investigate whether these changes are time-structured and how they relate to affective changes. In order to do so, linear AR models with Gaussian variables can be *generalized* to non-Gaussian variables (Grunwald, Hyndman, Tedesco, & Tweedie, 2000). In this case, I rely on a model proposed by Brandt & Sandler (2012). I introduce the original model along with the modified version in the following.

4.2 Model structures

4.2.1 A linear observation-driven Poisson autoregressive model

Brandt and Sandler (2012) propose a time-discrete linear vector autoregressive (VAR) model to capture the joint dynamics of multiple event count TS. As a univariate variant, the model equals

$$\eta_t = \beta Y_{t-1} + \zeta_t^* \tag{4.1}$$

$$\zeta_t^* = \exp(\alpha + \zeta_t) \tag{4.2}$$

with

$$\zeta_t \sim N(0, \psi),$$

where η_t is a continuous-valued latent stochastic process, which is regressed on a discrete-valued observed stochastic process Y_{t-1}, β is the corresponding lagged regression weight, and ζ_t^* is a continuous-valued latent stochastic process, which is the exponential function of a constant α plus a Gaussian white noise process ζ_t. The Gaussian white noise process consists of continuous-valued latent random variables $\{\zeta_t: t \in 1, \dots, T\}$ that are normally distributed with zero mean and variance ψ, over time mutually independent, and independent of the observed variables at all previous time points. It follows that the transformed random variables $\{\zeta_t^*: t \in 1, \dots, T\}$ are also mutually independent over time, independent of the observed variables at all previous time points, and lognormally distributed with location parameter α and scale parameter ψ or mean $\gamma = \exp(\alpha + 0.5\psi)$ and variance $\omega = (\exp(\psi) - 1)\exp(2\alpha + \psi)$ (Johnson, Kotz, & Balakrishnan, 1994).

A given latent state $\eta_t = \eta_t'$ realizes into an observed state via the Poisson distribution with mean η_t', hence, for each time point, the model postulates the following distributional assumption for the observed variables given a certain latent process state

$$Y_t| \, (\eta_t = \eta_t') \sim Pois(\eta_t'). \tag{4.3}$$

The latent process η_t thus represents changing event rates, while the observed process Y_t represents changing event counts. I therefore use the labels *event rate process* and *event count process* in the following. Restricting the novel input to the event rate process from the *event rate residual process* ζ_t^* to a non-negative state-space by applying the exponential function, increases the likelihood that the event rate process takes on values in accordance with the admissible range of the event rate parameter. So, a non-negative real number.[5]

I now propose the following modifications of the model: First, instead of using past event counts to predict future event rates, I change the model-structure so that the temporal dynamics unfold exclusively at the level of the latent event rate process. In the terms of the literature on non-Gaussian AR modeling, I propose to switch from an *observation-driven* to a *parameter-driven* structure (Cox, 1981; Davis, Dunsmuir, & Streett, 2003). Whereas observation-driven models are computationally cheaper and easier to forecast from, parameter-driven models entail a clear distinction between a latent process model and a measurement model, linking the latent process variables to observed indicators. This is associated with clearer interpretability (Brandt et al., 2000; Davis et al., 2003; Davis, Wang, & Dunsmuir, 1999). Specifically, one may argue that parameterizations of temporal patterns can only be meaningful, if they pertain to change in one and the same entity over time. This should hold especially in psychological applications, which are usually more interested in explanation than prediction (Yarkoni & Westfall, 2016). Likewise, parameter-driven formulations seem to be the norm in psychological adaptions of latent (Gaussian) TS models (e.g., Chow et al., 2010; Visser, 2011). Second, I want the model to account for event dynamics within multiple TS (i.e., individuals)

[5] Note that the transformation of the event rate residual process alone is neither necessary nor sufficient for an appropriate (i.e., non-negative) latent state-space and thus a defined model. A sufficient condition would be given by a non-negative state-space of the event rate residual process *and* a non-negative regression weight β. This condition is however not necessary, as the level of the event rate process may be high enough to prevent negative values even with a negative regression weight.

simultaneously. I therefore add a between-subject model that controls for stable differences between individuals and permits the estimation of average IA dynamics.

4.2.2 A linear parameter-driven Poisson autoregressive model

For a given individual i, the IA process model equals

$$\eta_{t,i} = \beta\eta_{t-1,i} + \zeta^*_{t,i} \tag{4.4}$$

$$\zeta^*_{t,i} = \exp((\alpha_i = \alpha'_i) + \zeta_{t,i}) \tag{4.5}$$

with

$$\zeta_{t,i} \sim N(0, \psi),$$

where $\eta_{t,i}$, β, $\zeta_{t,i}$, and $\zeta^*_{t,i}$ are defined as in the previous section, apart from the additional Person-index i and apart from $\{\alpha_i : i \in 1, \dots, N\}$ being continuous-valued latent random variables taking on person-specific values $\{\alpha'_i : i \in 1, \dots, N\}$. Also, this model assumes that the transformed and untransformed event rate residual process variables are independent of the event rate process at all previous time points. Again, the exponential function and a non-negative regression weight β are sufficient to ensure event rates that lead to defined event count predictions.

Analogous to the model component presented in Equation (4.3), event rates realize into event counts via the Poisson distribution, that is,

$$Y_{t,i} | \left(\eta_{t,i} = \eta'_{t,i}\right) \sim Pois\left(\eta'_{t,i}\right). \tag{4.6}$$

Unlike in the original implementation, however, one can now clearly think of Equation (4.6) as a measurement model that links unobservable event rates to observable event counts, and stands in contrast to a process model in Equations (4.4) and (4.5), which governs the transitions between the event rate process variables over time and thus accounts for change in the number of events a person is *expected* to experience in a defined period prior to measurement. In this setup, two sources of error can be distinguished, that is, latent process error, corresponding to the event rate residual process, and measurement error, introduced via the conditional Poisson distribution. Whereas latent process errors perturb the event rate process and uncover its dynamics over time, measurement errors do not affect future (or present) event rates, but only present event counts. One might thus speak of *Poisson measurement error*, although the additive decomposition that one usually sees with Gaussian measurement error (cf. Equation (2.2)) is not applicable here.

Differences between individuals are modeled as

$$\alpha_i = \kappa + \xi_i \tag{4.7}$$

with

$$\xi_i \sim N(0, \phi),$$

where the variables $\{\alpha_i : i \in 1, \dots, N\}$ decompose into a person-general constant κ, and continuous-valued latent random variables $\{\xi_i : i \in 1, \dots, N\}$, which are independent and normally distributed with zero mean and variance ϕ, and hence capture individual differences in the location of the event rate process after controlling for lagged relationships. Note that individual parameters $\alpha_i = \alpha_i'$ in the IA process model lead to individual process means *and* variances. Underlying this is the fact that in skewed distributions, the variance is a measure of dispersion that also carries information about the deviation of the mean from the other location parameters. If one, for instance, considers the geometric mean (i.e., the median, in case of the Lognormal distribution) and the geometric variance, then $\alpha_i = \alpha_i'$ has an effect only on the location but not on the dispersion of the long-run probability distribution of the process.

I shall finally describe the long-run behavior of the IA model. If the event rate process is stable (i.e., $|\beta| < 1$, implying that the temporal dependencies are not too strong and perturbations die out on the long-run), it has a weakly stationary long-run distribution (i.e., a time-invariant mean, and co-variances that only depend on the lag). The corresponding proof can be found in Appendix D. Per Wold's decomposition theorem, which applies to weakly stationary stochastic processes (Hamilton, 1994; Lütkepohl, 2005), the long-run distribution of the event rate process is a weighted Lognormal sum distribution (i.e., a weighted sum of the lognormally distributed event rate residual process variables, which becomes a sum of Lognormal distributions with different location parameters) with mean $\nu_i = (1-\beta)^{-1}\gamma_i$ and variance $\rho_i = (1-\beta^2)^{-1}\omega_i$. I derive these first two long-run moments in Appendix E. The long-run distribution of the event count process variables is a mixture of Poisson distributions – mixed according to the event rate processes' long-run distribution – with mean $\mu_i = \nu_i$ and variance $\sigma_i = \rho_i + \nu_i$ (cf. Karlis & Xekalaki, 2007).

Figure 4.2 shows Brandt and Sandlers' observation-driven PAR model (Panel A), my parameter-driven PAR model (Panel B), and a parameter-driven Gaussian AR model (Panel C) in terms of path diagrams to the left, model-implied trajectories in the model and long-run probability distributions to the right of each Panel. As the emphasis lies on the comparison of the different structures and their implications, I discard the between-subject component, and show simulated data for individual cases. From the trajectory segments it can be seen that the realization of the untransformed latent residual process $\zeta_{t,i}$ used to generate the data (grey solid line) is exactly the same across panels. But the assumed values play out differently in the different model structures. By comparing the trajectories of the latent and observed outcome processes $\eta_{t,i}$ and $Y_{t,i}$ (solid and dotted black lines) across panels, one notes a major difference between the observation-driven model and the two parameter-driven models. Whereas, in Panel A, the latent process to some extent follows earlier states of the manifest process, this is not the case in Panels B and C. Here, one observes more smoothly changing latent processes that are differently scaled but similarly structured over time, with manifest states realizing around, but not affecting (later) latent process states. On the long run, the two Poisson models behave more similar to each other than to the Gaussian model. That is, in Panels A and B, one observes right-skewed continuous-valued latent and discrete manifest distributions with non-negative support (i.e., over non-negative states only). The Gaussian model in Panel C, however, implies distributions that are both symmetric and continuous-valued at the latent and the manifest level and also assign non-zero densities to negative states.

Unless stated otherwise, I in the following refer to parameter-driven model structures exclusively.

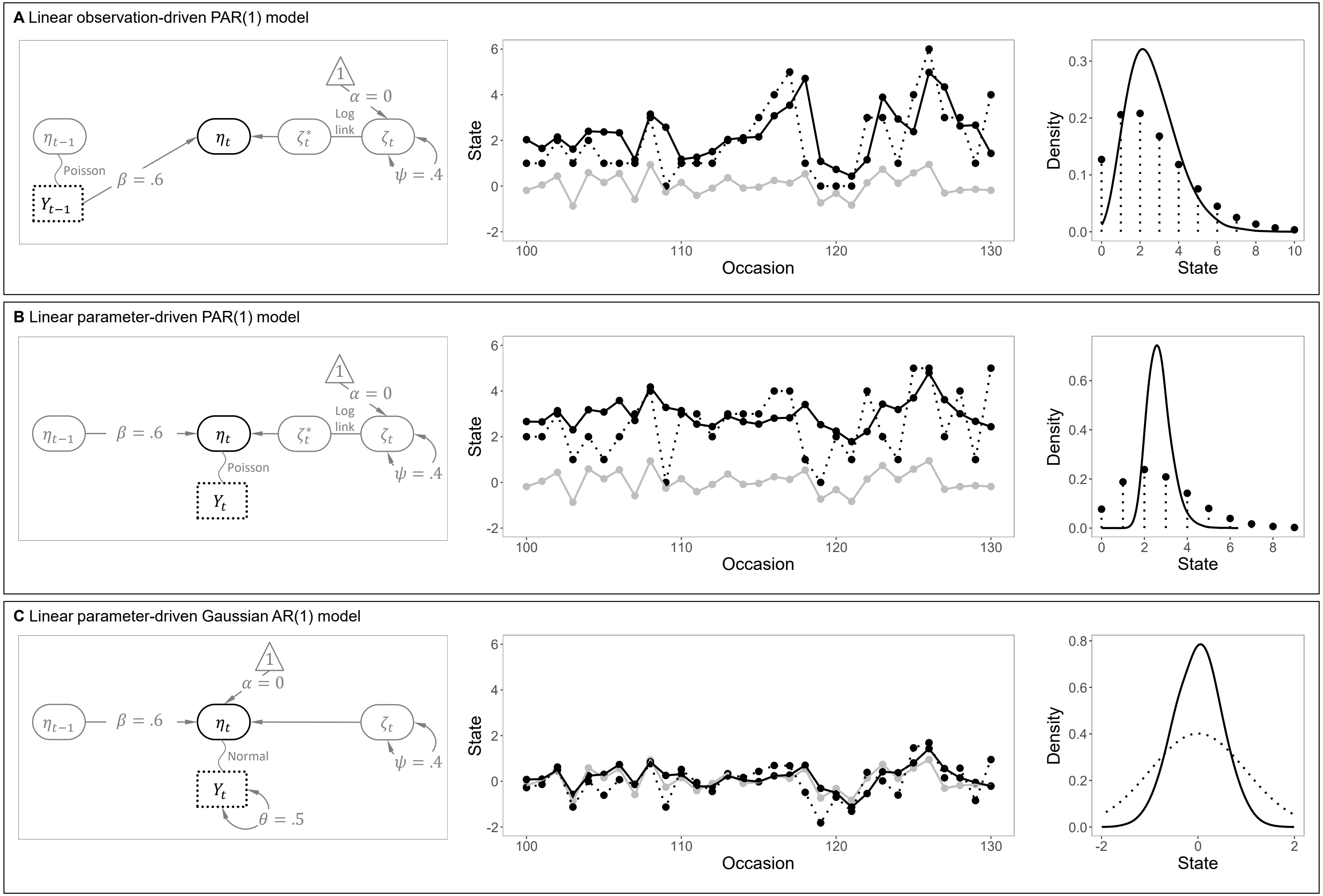

A Linear observation-driven PAR(1) model
η_{t-1}
Poisson
Y_{t-1}
$\beta = .6$
η_t
ζ_t^*
Log link
ζ_t
1
$\alpha = 0$
$\psi = .4$
State
Occasion
Density
State
B Linear parameter-driven PAR(1) model
η_{t-1}
$\beta = .6$
η_t
Poisson
Y_t
ζ_t^*
Log link
ζ_t
1
$\alpha = 0$
$\psi = .4$
State
Occasion
Density
State
C Linear parameter-driven Gaussian AR(1) model
η_{t-1}
$\beta = .6$
η_t
1
$\alpha = 0$
Normal
Y_t
$\theta = .5$
ζ_t
$\psi = .4$
State
Occasion
Density
State

Figure 4.2. The observation-driven PAR model, the parameter-driven PAR model, and a (parameter-driven) Gaussian AR model in terms of path diagrams, model-implied trajectories and long-run probability distributions. For the path diagrams, I partly adopt De Boeck and Wilson's (2004) scheme for item response models (i.e., the distribution of the observed variable given the latent variable is represented via a tilde, and the link function between the transformed and the untransformed process error are displayed). Additionally, observed variables are associated with dotted, and latent variables with solid lines. As I discard the between-subject component and show simulated data for single individuals, I drop the index *i* in this presentation. Across models, I used the same generating parameter values and the same random seeds for the generation of stochastic fluctuations over time, as this allows direct visual comparisons of model behaviors. The trajectory of the untransformed latent residual process, which is the source of all variations at the latent level, is displayed in grey. These fluctuations are exactly the same across panels, but play out differently across the different model structures. For the latent and observed outcome processes, the trajectory segments, and thus the models' implications for temporal patterns, are most distinct for the observation- versus parameter-driven model structures. The long-run distributions, on the other hand, are most distinct for the Poisson versus the Gaussian model structures.

4.2.3 A linear parameter-driven hybrid Poisson-Gaussian vector autoregressive model

I now extend the univariate model into a bivariate model, which for a given individual i equals

$$\boldsymbol{\eta}_{t,i} = \mathbf{B}\boldsymbol{\eta}_{t-1,i} + \boldsymbol{\zeta}^*_{t,i} \tag{4.8}$$

$$\boldsymbol{\zeta}^*_{t,i} = \exp\Big((\boldsymbol{\alpha}_i = \boldsymbol{\alpha}'_i) + \boldsymbol{\zeta}_{t,i}\Big) \tag{4.9}$$

with

$$\boldsymbol{\zeta}_{t,i} \sim N(\mathbf{0}, \boldsymbol{\Psi}),$$

where $\boldsymbol{\eta}_{t,i}$ is a 2×1 vector of continuous-valued latent stochastic processes, $\mathbf{B}$ is a 2×2 matrix of auto- and cross-lagged regression weights, $\{\boldsymbol{\alpha}_i : i \in 1, \dots, N\}$ are 2×1 vectors of continuous-valued latent random variables, $\{\boldsymbol{\alpha}'_i : i \in 1, \dots, N\}$ are 2×1 vectors of person-specific constants, and $\boldsymbol{\zeta}_{t,i}$ is a 2×1 vector of Gaussian white noise processes with zero mean vector and a 2×2 positive definite covariance matrix $\boldsymbol{\Psi}$. In analogy to the univariate model variant, it follows that the transformed residual process variables $\{\boldsymbol{\zeta}^*_{t,i} : t \in 1, \dots, T\}$ are mutually independent over time and lognormally distributed with mean vector $\boldsymbol{\gamma}_i$ and covariance matrix $\boldsymbol{\Omega}_i$, with entries $\gamma_{i_j} = \exp(\alpha'_{i_j} + 0.5\psi_{jj})$ and $\omega_{i_{jk}} = \exp\Big(\alpha'_{i_j} + \alpha'_{i_k} + 0.5\big(\psi_{jj} + \psi_{kk}\big)\Big)\big(\exp(\psi_{jk}) - 1\big)$ for $(j,k) \in (1,2)$, respectively (Ghasem, 2001). Both the untransformed and transformed residual process variables are independent of the event rate process at all previous time points.

The measurement model encompasses two equations, namely,

$$Y_{E_{t,i}} | \big(\eta_{E_{t,i}} = \eta'_{E_{t,i}}\big) \sim Pois\big(\eta'_{E_{t,i}}\big) \tag{4.10}$$

and

$$Y_{A_{t,i}} | \big(\eta_{A_{t,i}} = \eta'_{A_{t,i}}\big) \sim N\big(\eta'_{A_{t,i}}, \theta\big), \tag{4.11}$$

where $\eta_{E_{t,i}}$, the first element in $\boldsymbol{\eta}_{t,i}$, is the event rate process, with predicted event rates $\eta'_{E_{t,i}}$ realizing into event counts via the Poisson distribution with mean $\eta'_{E_{t,i}}$, and where $\eta_{A_{t,i}}$, the second element in $\boldsymbol{\eta}_{t,i}$, is the latent affect process, with predicted latent affective states $\eta'_{A_{t,i}}$ realizing into observed affective states via the normal distribution with mean $\eta'_{A_{t,i}}$ and variance θ. Note that only by virtue of the measurement models, a distinction between the two latent processes is introduced. This also implies a potentially skewed distribution of the affective states, which, however, seems in line with the typically observed distributional shape of negative affect (e.g., Schilling & Diehl, 2014). Assuming a transformed latent residual process only for the event rate process would be problematic in that the latent affect residual process also feeds into the event rate process if there is a non-zero cross-regressive (CR) effect, potentially leading to inadmissible event rates.

In analogy to Equation (4.7), the between subject model equals

$$\boldsymbol{\alpha}_i = \boldsymbol{\kappa} + \boldsymbol{\xi}_i \tag{4.12}$$

with

$$\boldsymbol{\xi}_i \sim N(\mathbf{0}, \boldsymbol{\Phi}),$$

where the variables $\{\boldsymbol{\alpha}_i : i \in 1, \dots, N\}$ decompose into a 2×1 vector of person-general constants $\boldsymbol{\kappa}$, and 2×1 vectors of continuous-valued latent random variables $\{\boldsymbol{\xi}_i : i \in 1, \dots, N\}$, which are independent and normally distributed with zero mean vector and a 2×2 positive definite covariance matrix $\boldsymbol{\Phi}$.

Finally, if the process is stable (i.e., all eigenvalues of the matrix Beta have modulus less than one; Lütkepohl, 2005), it has a weakly stationary long-run distribution, assuming that the equivalence between the univariate Poisson and Gaussian AR model shown in Appendix D also extends to the multivariate case. This is again a weighted Lognormal sum distribution with mean vector $\boldsymbol{\nu}_i = (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\gamma}_i$ and vectorized covariance matrix $\mathrm{vec}(\mathbf{P}_i) = \big(\mathbf{I} \otimes \mathbf{I} - (\mathbf{B} \otimes \mathbf{B})\big)^{-1} vec(\boldsymbol{\Omega}_i)$, where $\mathbf{I}$ is a 2×2 identity matrix, $\otimes$ denotes the Kronecker product and $\mathrm{vec}(\boldsymbol{\Omega}_i)$ is the vectorization of $\boldsymbol{\Omega}_i$. I derive these first two long-run moments in Appendix F. At the manifest level, the model implies again a mixture of Poisson distributions for $Y_{E_{t,i}}$, with mean $\mu_{E_i} = \nu_{E_i}$ and variance $\sigma_{E_i} = \rho_{E_i} + \nu_{E_i}$, and an infinite mixture of normal distributions for $Y_{A_{t,i}}$.

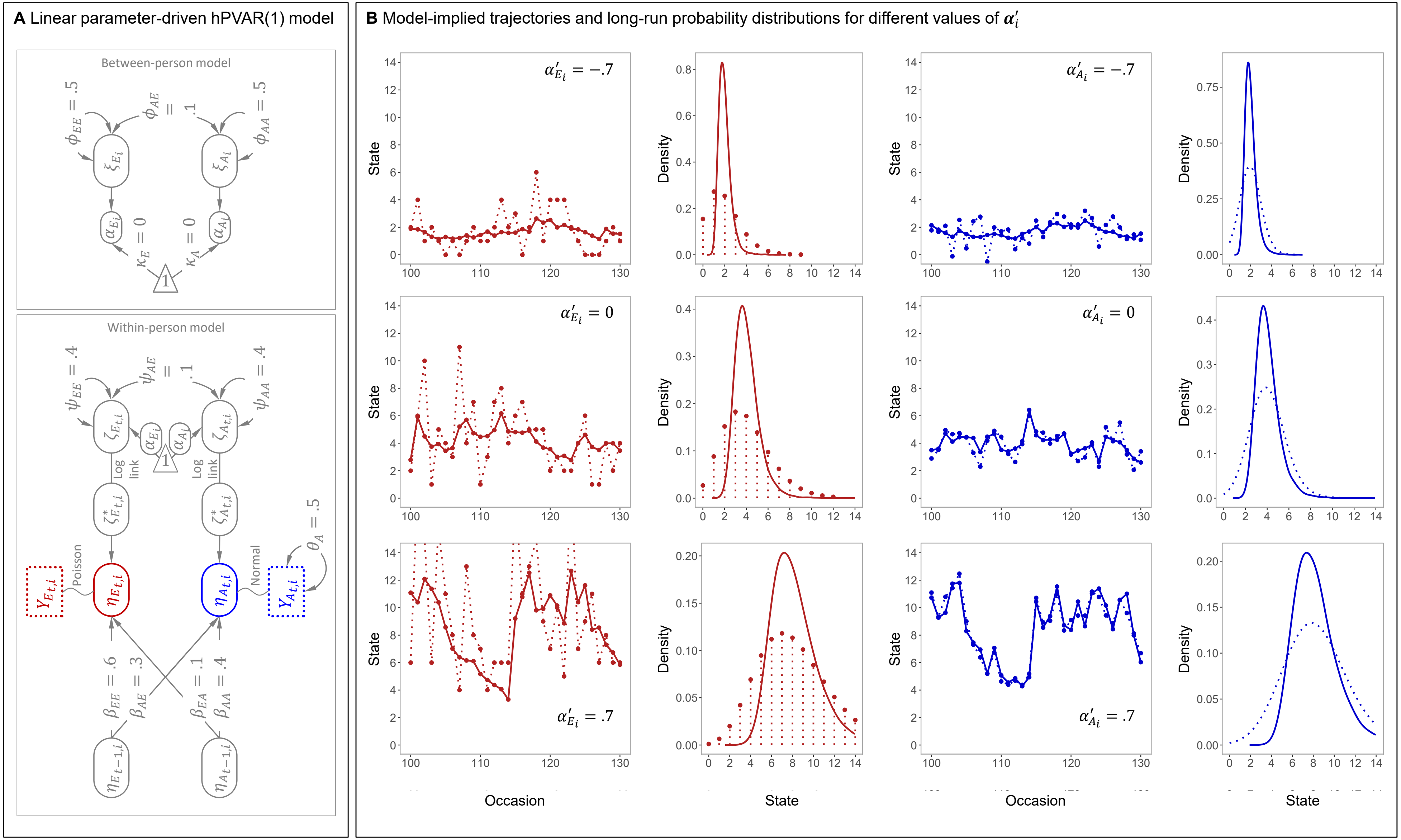

A Linear parameter-driven hPVAR(1) model
Between-person model
Within-person model
Log link
Poisson
Normal
B Model-implied trajectories and long-run probability distributions for different values of α'_i
$\alpha'_{E_i} = -.7$
$\alpha'_{A_i} = -.7$
$\alpha'_{E_i} = 0$
$\alpha'_{A_i} = 0$
$\alpha'_{E_i} = .7$
$\alpha'_{A_i} = .7$
State
Density
Occasion

Figure 4.3. The linear parameter-driven hPVAR model for event counts and affective states in terms of a path diagram, model-implied trajectories, and long-run density distributions. For the path diagrams, I partly adopt De Boeck and Wilson’s (2004) scheme for item response models (i.e., the distribution of the observed variable given the latent variable is represented via a tilde, and the link function between the transformed and the untransformed process error are displayed). Additionally, observed variables are associated with dotted, and latent variables with solid lines, and events are coded in red, while affect is coded in blue. Trajectory segments and corresponding long-run distributions are displayed for three different hypothetical individuals with distinct values for the conditional location parameter of the latent process. The displayed individuals thus fluctuate around different levels, as reflected in the trajectory segments, resulting in the different locations of the long-run distributions. Since the perturbations of the process errors scale with the size of the conditional location parameter due to the exponential function (applied to the sum of the intercept and stochastic residual), individuals also stably differ in the amount of fluctuations and thus the dispersions of the long-run distributions under this parameterization.

Figure 4.3 shows hPVAR model for event counts and affective states in terms of a path diagram (Panel A) and process-specific model-implied trajectories and long-run distributions (Panel B). Trajectory segments and corresponding long-run distributions are displayed for three different hypothetical individuals with distinct values for the person-specific conditional location parameter $\boldsymbol{\alpha}_i'$ for both processes. The implied stable differences between individuals are directly apparent. These concern the levels around which the latent processes fluctuate and thus the locations of the long-run distributions. All else equal and as in the univariate model, they also concern the amount of fluctuations the processes display and thus the dispersions of the long-run distributions. This is because the realizations of the process errors scale with the size of the location parameter due to the exponential function applied to the process errors of both processes. Note that a process showing little fluctuations around a high level could in principle also be captured by this model and would be implied by a high location parameter in combination with a smaller process residual variance.

4.3 Model estimation

To fit the model to data, I rely on simulation-based estimation via MCMC methods in the Bayesian framework (for a general treatment see, e.g., Gelman, Carlin, Stern, & Rubin, 2004; Jackman, 2009). In Bayesian modeling, inference is based on the posterior probability distribution of the model parameters (also denoted posterior in the following). The posterior is, by Bayes theorem, a joint probability distribution that is proportional to the product of the likelihood of the data under the probability model for the data, and the prior probability distribution for the model parameters (also denoted prior in the following). The explication of uncertainty in *all* model parameters via probability distributions a priori to the analysis, and hence, the incorporation of all model parameters as random variables into the analysis, regardless of the hierarchical structure of the data, is a major difference between Bayesian and classical frequentist statistics (e.g., Wasserman, 2004). Updating the priors by the likelihood of the data under the model yields posterior probabilities and thus plausible parameter values given the particular data set.

Simulation-based estimation, and specifically MCMC methods allow working with complicated, high-dimensional posterior distributions for which analytical or numerical solutions are not available or difficult to obtain (Stern, 1997). This affords high flexibility in modeling and contributes to the popularity of the approach. The posterior distribution is approximated by drawing samples from it. Here, I rely on Gibbs sampling as a specific MCMC technique (Geman & Geman, 1984). Gibbs sampling recovers the unknown joint posterior by drawing from the known conditional posteriors of each variable involved. Once a sufficient number of samples is acquired (i.e., convergence criteria are met), descriptive statistics of the approximated distribution serve as Bayesian estimators (e.g., marginal means and modes as point estimates, marginal quantiles as interval estimates).

I present the posterior distributions associated with the portrayed models in the following. I also describe the implementation of the estimation procedure in terms of software. The presented settings generalize to the following simulations and real data applications, if not stated otherwise.

4.3.1 Posterior distribution for the Poisson autoregressive model

For the PAR model, the posterior probability distribution for the model parameters, marginalized over the time-varying latent states, is

$$f(\alpha_i, \kappa, \phi, \beta, \psi | \mathbf{y}_i) \propto f(\mathbf{y}_i | \alpha_i, \beta, \psi) f(\alpha_i | \kappa, \phi) f(\kappa, \phi\ \beta, \psi) \tag{4.13}$$

As can be seen above, the posterior distribution is proportional to the product of three probability distributions.

The first one is the likelihood function of the data given the model parameters. For a given time point t the likelihood function equals

$$f\left(\mathrm{y}_{t,i} | \eta_{t-1,i}, \alpha_i, \beta, \psi\right) = \int_0^\infty f\left(Y_{t,i} |\ \eta_{t,i}\right) f\left(\eta_{t,i} | \eta_{t-1,i}, \alpha_i, \beta, \psi\right) \mathrm{d}\eta_{t,i}, \tag{4.14}$$

where

$$f\left(Y_{t,i} |\ \eta_{t,i}\right) = Pois\left(Y_{t,i}; \eta_{t,i}\right),$$
$$f\left(\eta_{t,i} | \eta_{t-1,i}, \alpha_i, \beta, \psi\right) = lnN\left(\eta_{t,i}; \alpha_i, \psi\right)\ shifted\ by\ \beta\eta_{t-1,i}.$$

Conditioning on $\eta_{t-1,i}$ takes the temporal structure of the event rate process into account and renders the individual likelihoods mutually independent over time.

The remaining distributions are the prior distribution for the IA conditional location parameters

$$f(\alpha_i | \kappa, \phi), \tag{4.15}$$

where

$$f(\alpha_i|\kappa,\phi) = N(\alpha_i;\kappa,\phi),$$

and, the hyperprior distributions for the IA conditional location parameters, and prior distributions for the remaining model parameters

$$f(\kappa,\phi,\beta,\psi) = f(\kappa)f(\phi)f(\beta)f(\psi), \tag{4.16}$$

where

$$\begin{aligned} f(\kappa) &= N(\kappa; m_\kappa, v_\kappa), \\ f(\phi) &= \Gamma^{-1}\left(\phi; s_\phi, r_\phi^{-1}\right), \\ f(\beta) &= N\left(\beta; m_\beta, v_\beta\right), \\ f(\psi) &= \Gamma^{-1}\left(\psi; s_\psi, r_\psi^{-1}\right). \end{aligned}$$

The above distributions are parameterized as follows: As previously, the Poisson distribution is parameterized by a rate parameter, the Lognormal distribution is parameterized by a location and a scale parameter, and the normal distribution is parameterized by a mean and a variance. I additionally introduce the inverse Gamma distribution, which is the conjugate prior for the variance of a normal distribution, parameterized by a shape and an inverse rate. Greek letters indicate unknown quantities that are freely estimated within the model, Latin letters indicate quantities that are fixed to specific values, such as the parameters shaping the (hyper-)priors in Equation (4.16).

Note that, so far, I did not indicate how the latent process variables at the initial time point, $\eta_{t=0,i}$ feature in the likelihood function. In the present context, I rely on the conditional likelihood function, that is, per individual, I equate the initial latent states to the model-implied long-run process means by setting up the corresponding parameter constraints.

4.3.2 Posterior distribution for the hybrid Poisson-Gaussian vector autoregressive model

For the hPVAR model, the posterior probability distribution for the model parameters, marginalized over the latent states, is

$$f(\boldsymbol{\alpha}_i, \boldsymbol{\kappa}, \boldsymbol{\Phi}, \mathbf{B}, \boldsymbol{\Psi}, \theta | \mathbf{y}_i) \propto f(\mathbf{y}_i | \boldsymbol{\alpha}_i, \mathbf{B}, \boldsymbol{\Psi}, \theta) f(\boldsymbol{\alpha}_i | \boldsymbol{\kappa}, \boldsymbol{\Phi}) f(\boldsymbol{\kappa}, \boldsymbol{\Phi}, \mathbf{B}, \boldsymbol{\Psi}, \theta) \tag{4.17}$$

As for the univariate model, the posterior distribution is proportional to the product of three probability distributions.

Again, there is the likelihood function, which for a given time point t equals

$$\begin{aligned} &f\left(\mathbf{y}_{t,i} | \boldsymbol{\eta}_{t-1,i}, \boldsymbol{\alpha}_i, \mathbf{B}, \boldsymbol{\Psi}, \theta\right) \\ &= \int_0^\infty f\left(Y_{A,t,i} | \, \eta_{A,t,i}, \theta\right) f(Y_{E,t,i} | \, \eta_{E,t,i}) f\left(\boldsymbol{\eta}_{t,i} \middle| \boldsymbol{\eta}_{t-1,i}, \boldsymbol{\alpha}_i, \mathbf{B}, \boldsymbol{\Psi}\right) \, \mathrm{d}\boldsymbol{\eta}_{t,i}, \end{aligned} \tag{4.18}$$

where

$$\begin{aligned} f\left(y_{A,t,i} | \, \eta_{A,t,i}, \theta\right) &= N\left(y_{A,t,i}; \, \eta_{A,t,i}, \theta\right), \\ f\left(y_{E,t,i} | \, \eta_{E,t,i}\right) &= Pois\left(y_{E,t,i}; \eta_{E,t,i}\right), \\ f\left(\boldsymbol{\eta}_{t,i} | \boldsymbol{\eta}_{t-1,i}, \boldsymbol{\alpha}_i, \mathbf{B}, \boldsymbol{\Psi}\right) &= lnN\left(\boldsymbol{\eta}_{t,i}; \boldsymbol{\alpha}_i, \boldsymbol{\Psi}\right) \, shifted \; by \; \mathbf{B}\boldsymbol{\eta}_{t-1,i}, \end{aligned}$$

a prior distribution for the IA conditional location parameters

$$f(\boldsymbol{\alpha}_i | \boldsymbol{\kappa}, \boldsymbol{\Phi}) = f(\boldsymbol{\alpha}_i | \boldsymbol{\kappa}, \boldsymbol{\Phi}), \tag{4.19}$$

where

$$f(\boldsymbol{\alpha}_i | \boldsymbol{\kappa}, \boldsymbol{\Phi}) = N(\boldsymbol{\alpha}_i; \boldsymbol{\kappa}, \boldsymbol{\Phi}),$$

and hyperprior distributions for the IA conditional location parameters, and prior distributions for the remaining model parameters

$$f(\boldsymbol{\kappa}, \boldsymbol{\Phi}, \mathbf{B}, \boldsymbol{\Psi}, \theta) = f(\boldsymbol{\kappa}) f(\boldsymbol{\Phi}) f\left(\mathrm{vec}(\mathbf{B})\right) f(\boldsymbol{\Psi}) f(\theta), \tag{4.20}$$

where

$$\begin{aligned} f(\boldsymbol{\kappa}) &= N(\boldsymbol{\kappa}; \mathbf{m}_{\boldsymbol{\kappa}}, \mathbf{V}_{\boldsymbol{\kappa}}), \\ f(\boldsymbol{\Phi}) &= W^{-1}\left(\boldsymbol{\Phi}; \boldsymbol{S}_{\boldsymbol{\Phi}}^{-1}, df\right), \\ f\left(\mathrm{vec}(\mathbf{B})\right) &= N(\mathrm{vec}(\mathbf{B}); \mathbf{m}_{\mathbf{B}}, \mathbf{V}_{\mathbf{B}}), \\ f(\boldsymbol{\Psi}) &= W^{-1}(\boldsymbol{\Psi}; \; \mathbf{S}_{\boldsymbol{\Psi}}^{-1}, df), \\ f(\theta) &= \Gamma^{-1}\left(\theta; s_\theta, r_\theta^{-1}\right). \end{aligned}$$

The above distributions are parameterized as follows: The multivariate Lognormal distribution is parameterized by a location vector and a scale matrix, the multivariate normal distribution is parameterized by a mean vector and a covariance matrix, and the inverse Wishart distribution, which is the conjugate prior for the covariance matrix of a normal, is parameterized by an inverse scale matrix, and degrees of freedom equal to the dimensionality of the latent process (i.e., two in this case). Again, Greek letters indicate unknown quantities, Latin letters indicate quantities that are fixed to specific values. As for the univariate model, I rely on the conditional likelihood function for model estimation.

4.3.3 Markov chain Monte Carlo implementation

For model estimation, I use the free and open-source software JAGS (version 4.0.0; Plummer, 2003) controlled from R (version 3.3.2; R Core Team, 2016) via the R package rjags (Plummer, 2016). I sample from the posterior by evoking multiple parallel Markov chains. The sampling algorithms generating these chains are chosen automatically. For each chain and parameter, I provide stochastic starting values in plausible regions of the parameter space. Samplers are given a potentially long initial adaptation phase (up to, e.g., 30,000 iterations), during which they adjust their behavior to the specific data- and model- situation to improve sampling efficiency. This is followed by a high number of iterations (e.g., 330,000), of which I discard a first proportion as burn-in samples (e.g., 30,000). I thin by a factor of X (e.g., $X = 300$), meaning that only each X^{th} sample is saved. As Markov chains are characterized by temporal dependencies, sampled values contain less redundant information, the more they lie temporally apart. Eventually, inferences may rely on the remaining samples across all chains (e.g., 4000 samples). However, the *effective* sample size may be lower. How much information is available per parameter is thus indicated by the number of *effective samples*, a sample size estimate controlled for the auto-correlation that remains in the chains after thinning (Gelman et al., 2004).

4.4 Simulation study

The models I propose here are supposed to provide descriptions of the dynamics of daily stressors and the joint dynamics of stressors and affective experiences that generalize beyond a particular data set. I am therefore interested in the performance of the proposed Bayesian estimates in "repeated practical use" (Bayarri & Berger, 2004, p. 60). Note that, while frequentist estimates are defined by their long-run behavior over repeated (hypothetical) use, Bayesian estimates do not necessarily come with certain frequency properties (Gelman & Shalizi, 2013; Rubin, 1984; Wasserman, 2004, 2012). The main purpose of this simulation study is thus to investigate the frequentist performance of the PAR model and the hPVAR model under realistic settings.

To this end, I choose one set of what I deem plausible parameter values and investigate the accuracy and efficiency of Bayesian point and interval estimates over a number of simulated data replications. Realistic settings also involve finite data and I therefore vary the number of time points and individuals, looking at small to medium magnitudes for both. For selected sample size conditions, I include performance comparisons with the ordinary Gaussian counterparts of the Poisson models (i.e., Gaussian AR and VAR models of first order).

As a second part of this simulation, I present results regarding the performance of Gaussian AR and VAR models fitted to data generated from the PAR and the hPVAR model, respectively. This is supposed to probe the robustness of the Gaussian models under the present conditions and provide a justification for incorporating the Poisson distribution when modeling event TS.

4.4.1 Data- and model-conditions

I use the PAR and hPVAR model structures as presented in Equations (4.4) to (4.7) and Equations (4.8) to (4.12) for data generation. The parameters are fixed to the values presented in Figure 4.2, Panel B, and Figure 4.3, as these values were selected to produce face-valid, realistic data. In the univariate case, inter-individual (IE) variability in conditional locations, as captured by parameter ϕ, is fixed to 0.4. For data generation from the Gaussian models, I use the same parameter values and random intercept extensions of the structures presented in Equations (2.1) and (2.2) and Equations (3.1) and (3.2).

To create different finite data settings, I cross small to medium numbers of time points ($T = (30, 50)$) with small to medium numbers of persons ($N = (50, 100)$), and fit the respective true models under one to all four of these conditions.

To investigate the robustness of the Gaussian model structures to misspecification, I add five conditions with $N = 50$ and $T = 30$, in which I fit a Gaussian AR and a Gaussian VAR model to data generated under the PAR and the hPVAR model, respectively. For the univariate case, I additionally vary skewness of the latent and observed processes' long-run distributions by varying the average conditional location parameter κ, as displayed across lines in Figure

4.3, Panel B. The expectation here is that ordinary (V)AR models featuring the symmetrical normal distribution at both the latent and the manifest level should be less appropriate and should thus perform worse, the more skewed the distribution of the data. To follow up on this, I also test whether including just the transformed (i.e., skewed) latent process error (cf. Equations (4.4) and (4.5)), in combination with Gaussian measurement error, makes up for the full PAR model structure, and hence improves performance independent of the Poisson distribution.

4.4.2 Model building and fitting

The models fitted correspond to the data generating ones, so, again, the structures as presented in Equations (4.4) to (4.7), Equations (4.8) to (4.12), and between-subject extensions of the structures presented in Equations (2.1) and (2.2), and Equations (3.1) and (3.2).

For the hPVAR model, I modify the original latent process model, in that I further model the multivariate normal distribution of the untransformed process errors in $\boldsymbol{\zeta}_{t,i}$ as

$$\boldsymbol{\zeta}_{t,i} = \boldsymbol{\Delta}\iota_{t,i} + \boldsymbol{\Upsilon}_{t,i} \tag{4.21}$$

with

$$\iota_{t,i} \sim N(0,1),$$
$$\boldsymbol{\Upsilon}_{t,i} \sim N(\mathbf{0}, \mathbf{I}\pi),$$

where $\iota_{t,i}$ is a standardized Gaussian white noise process, functioning as a *common* process to the processes in $\boldsymbol{\zeta}_{t,i}$, and as such accounting for their covariation within time points, $\boldsymbol{\Delta}$ is a 2×1 matrix of loadings of the processes in $\boldsymbol{\zeta}_{t,i}$ on the common process, and $\boldsymbol{\Upsilon}$ is a 2×1 vector of Gaussian white noise processes with zero mean vector and a 2×2 positive definite diagonal covariance matrix $\mathbf{I}\pi$ with equal diagonal entries π (i.e., equal variances) [6].

This parameterization implies a further decomposition of the process error's original covariance matrix $\boldsymbol{\Psi}$ (cf. Equation (4.9)) into $\boldsymbol{\Psi} = \boldsymbol{\Delta}\boldsymbol{\Delta}^{\mathrm{T}} + \mathbf{I}\pi$. In terms of its likelihood, this model is mathematically equivalent to the original model. I still use this alternative parameterization, as it resolved adaption problems the JAGS samplers had otherwise.

[6] Note that estimating different loadings – instead of different residual variances – allows for a negative covariance between the two outcome processes.

To estimate the models, the parameters of the (hyper-)prior distributions have to be set. I aim at relatively vague (hyper-)priors to minimize bias. In case of the PAR model, I set priors, with distributional moments as presented in Chapter 4, Sections 0 and 4.3.2, to

$$f(\kappa) = N(\kappa; 0{,}100),$$
$$f(\phi) = \Gamma^{-1}(\phi; .001, .001),$$
$$f(\beta) = N(\beta; 0{,}100),$$
$$f(\psi) = \Gamma^{-1}(\psi; .001, .001).$$

For the Gaussian AR model, I add

$$f(\theta) = \Gamma^{-1}(\theta; .001, .001).$$

Normal distributions with means of 0 and variances 100 are relatively wide, and inverse Gamma distributions with shapes and inverse rates of .001 are a default choice for flat variance priors (Gelman, 2006).

In case of the Gaussian VAR model, I specify priors as

$$f(\boldsymbol{\kappa}) = N\left(\boldsymbol{\kappa}; \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}100 & 0\\0 & 100\end{bmatrix}\right),$$

$$f(\boldsymbol{\Phi}) = W^{-1}\left(\boldsymbol{\Phi}; \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, 2\right),$$

$$f(\text{vec}(\mathbf{B})) = N\left(\text{vec}(\mathbf{B}); \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}100 & 0 & 0 & 0\\0 & 100 & 0 & 0\\0 & 0 & 100 & 0\\0 & 0 & 0 & 100\end{bmatrix}\right),$$

$$f(\boldsymbol{\Psi}) = W^{-1}\left(\boldsymbol{\Psi}; \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, 2\right),$$

$$f(\theta_k) = \Gamma^{-1}(\theta_k; .001, .001)\ for\ k \in (1,2).$$

For the alternatively parameterized hPVAR model, I exchange $f(\boldsymbol{\Psi})$ for

$$f(\boldsymbol{\iota}) = N\left(\boldsymbol{\iota}; \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}100 & 0\\0 & 100\end{bmatrix}\right),$$

$$f(\pi) = \Gamma^{-1}(\pi; .001, .001),$$

and reduce $f(\theta_k)$ to

$$f(\theta) = \Gamma^{-1}(\theta; .001, .001).$$

Again, I rely on relatively wide multivariate normal distributions, and a common parameterization of the inverse Wishart (Schuurman, Grasman, et al., 2016), and the inverse Gamma distribution.

The simulation of the posterior distributions relies on different amounts of samples across conditions. For the univariate models, the numbers of adaptive, burn-in, and final samples are as described in Chapter 4, Section 4.3.3. For the bivariate models, I increase these numbers to ensure sufficiently high convergence rates. In case of the hPVAR model, I double the number of burn-in samples to 60,000, and in case of the VAR model, I use 100,000 burn-in and 500,000 final samples with a thinning factor of 500.

Estimating the different models under this set up (i.e., running one replication) on the cluster computing system of the Max Planck Institute for Human Development (Intel® Xeon® Processor E5-2670 processors, using parallel cores for running the separate Markov chains) takes at shortest around half an hour and at longest around two days.

Per simulation condition, I aim at 500 converged replications of the univariate models and 250 converged replications of the bivariate models[7].

4.4.3 Performance criteria

First, as I present results from converged solutions only, a criterion to decide whether models have sufficiently converged is required. I primarily rely on the potential scale reduction factor $\hat{R}$ (Gelman & Rubin, 1992). Per sampled parameter, $\hat{R}$ represents a quantification of the total variance within and between Markov chains over the variance within chains. Under convergence, this ratio should be close to one, indicating that the relative variance between chains is negligible and most variation lies within chains. I infer *sufficient* convergence only if the $\hat{R}$ point estimate is below a value of 1.1 for all parameters per model, and drop replications for which this is not the case. I additionally ensure by visual inspection that $\hat{R}$ converges to one with increasing chain length. Also, I check that the traces of the Markov chains (i.e., the samples I base my inference on) appear sufficiently stationary and mixed per model parameter. I inspect $\hat{R}$ plots and traceplots for a random subset of 10 solutions per condition. Finally, I report when samplers could not adapt to the specific data- and model-situation during the initial adaption phase. Of the converged solutions, I exclude replications with less than 50 efficient samples for any of the parameters.

I present model solutions in terms of the following summary statistics of the posterior distribution: Of the marginal posterior distribution per parameter, I use the means (i.e., the expected a posteriori estimator or EAP estimator) and the modes (i.e., the maximum a posteriori estimator, or MAP estimator) as point estimates, and the 2.5th and 97.5th percentiles as interval estimates (i.e., Bayesian credible intervals; BCIs). Whereas the EAP is a prominent estimator in Bayesian statistics, the MAP may be of interest as it is equivalent to the maximum likelihood (ML) estimator given flat priors (Karch, 2016). Also, for skewed distributions, the mode may be a more specific measure of the distributions' central tendency than the mean. I obtain modes via kernel density estimation methods as implemented in the R package modeest (Poncet, 2012).

To evaluate the performance of the point estimates, I report absolute and relative bias as a measure of accuracy and the root-mean-square error (RMSE) as a measure that combines

[7] Exemplary resampling within a condition with 500 converged replications reveals that simulation results are relatively stable even with only 250 replications.

accuracy and efficiency. Relative bias is calculated as absolute bias relative to the true parameter value in percent, the RMSE is calculated as the square root of the sum of the estimate's variance across replications and its squared absolute bias. I also report statistical significance of absolute and relative bias according to two-tailed t-Tests ($\alpha = .05$, uncorrected).

Analogously, to evaluate the performance of the interval estimates, I report coverage rates as a measure of accuracy and interval width averaged across replications as a measure of efficiency. Coverage rates are calculated as the percentage of replications in which intervals contain the true parameter value. For all parameters with non-zero true values, I additionally report the proportion of interval estimates in percent that exclude zero and have support of appropriate sign, reflecting power to detect effects as different from zero in the right direction.

For checking convergence and summarizing the posterior distribution, I mainly make use of the functionality of the R package coda (Plummer, Best, Cowles, & Vines, 2006). Following recommendations given in the literature, I refer to biases above 10 %, and to coverage rates below 90 % as substantial (e.g., Grund, Lüdtke, & Robitzsch, 2017; Schafer & Graham, 2002). Smaller RMSEs and narrower intervals, given acceptable coverage rates, are preferable.

4.4.4 Results with discussion

Table 4.1 displays the results for the PAR model under all sample size conditions (i.e., for all combinations of N and T in Sections A to D). Across conditions, there were neither problems with non-convergence due to $\hat{R}$ values being above the cut-off of 1.1 or numbers of effective samples being below the cut-off of 50, nor did samplers show adaption problems. Traceplots show chains that appear relatively stationary and well-mixed. The plotted series of $\hat{R}$ estimates show global convergence towards one with increasing chain length, including occasional local distortions on a small scale.

Performance is relatively good across all sample size conditions. Biases are, although sometimes statistically significant, of non-substantial size (i.e., below 10 %) for both the EAP and the MAP estimator. Also, coverage rates of the BCIs seem generally acceptable (i.e., near 95 %). A more detailed look at these criteria reveals that, with increasing N and increasing T, the biases and RMSEs of the EAP and, less pronounced, the MAP estimator decrease, while the BCI coverage rates show slight improvements and interval widths reduce.

Table 4.1. Performance of the PAR model fitted to data generated from the PAR model

Parameter		EAP			MAP			BCI 95%			Convergence	
Name	True	Bias	Bias%	RMSE	Bias	Bias%	RMSE	Cover	Width	Power	Rhat	Neff
A: $N = 50, T = 30$												
κ	0.00	-0.01	-	0.23	0.01	-	0.23	94.40	0.90	-	1.00	1443.44
β	0.60	-0.01*	-1.43*	0.07	0.00	-0.19	0.07	92.80	0.25	99.60	1.00	1569.73
ψ	0.40	0.03*	6.88*	0.11	-0.01	-1.78	0.10	93.20	0.39	100.00	1.00	1705.99
ϕ	0.50	0.02*	4.76*	0.12	-0.02*	-3.75*	0.11	94.60	0.46	100.00	1.00	4046.26
B: $N = 100, T = 30$												
κ	0.00	-0.01	-	0.16	0.01	-	0.16	94.80	0.62	-	1.00	1500.30
β	0.60	-0.01*	-1.15*	0.05	0.00	-0.51	0.05	96.00	0.18	100.00	1.00	1595.99
ψ	0.40	0.01*	3.05*	0.07	0.00	-0.92	0.07	94.40	0.26	100.00	1.00	1772.28
ϕ	0.50	0.01*	1.89*	0.07	-0.01*	-2.32*	0.07	96.40	0.31	100.00	1.00	4046.10
C: $N = 50, T = 50$												
κ	0.00	0.01	-	0.17	0.02*	-	0.17	95.40	0.69	-	1.00	1564.28
β	0.60	-0.01*	-1.60*	0.05	-0.01*	-0.93*	0.05	95.80	0.18	100.00	1.00	1843.09
ψ	0.40	0.00	0.68	0.06	-0.01*	-2.84*	0.06	95.00	0.25	100.00	1.00	2082.51
ϕ	0.50	0.02*	3.99*	0.11	-0.02*	-4.21*	0.10	95.60	0.44	100.00	1.00	4041.02
D: $N = 100, T = 50$												
κ	0.00	-0.01	-	0.12	0.00	-	0.12	94.60	0.48	-	1.00	1596.87
β	0.60	0.00	-0.45	0.03	0.00	-0.13	0.03	96.00	0.13	100.00	1.00	1850.69
ψ	0.40	0.01*	1.52*	0.04	0.00	-0.34	0.04	95.60	0.18	100.00	1.00	2098.89
ϕ	0.50	0.01*	1.63*	0.08	-0.01*	-2.35*	0.07	94.40	0.30	100.00	1.00	4051.62

Note. True = true value, used for data generation; Cover = coverage in %; power in %; Rhat = $\hat{R}$; Neff = number of effective samples; * p < (.05); absolute values below or equal to .005 are rounded to 0. The number of replications is 500 per condition A - D.

These trends might reflect the diminishing influence of the employed prior distributions and/or the reduction of (other) finite sample biases with increasing amounts of data. Power is at ceiling throughout.

Within conditions, the pattern of biases is such that parameters that reflect variance components are consistently underestimated by the MAP and overestimated by the EAP estimator, while the AR effect β is consistently underestimated by both. The present models feature AR effect-prior distributions that are centered at zero, possibly introducing this bias. In addition, a finite sample bias towards zero in (Gaussian) AR modeling has been described for the ordinary least squares estimator (Maeshiro, 2000), and for the ML estimator (Cheang & Reinsel, 2000; Marriott & Pope, 1954). Bayesian estimators of AR effects might thus also be affected to the degree the assigned prior distributions possess a flat shape.

The overestimation of variances by the EAP estimator might stem from the fact that the corresponding priors are restricted with respect to negative values, but unrestricted with respect to positive values (i.e., positive values are admissible, negative values are not; Gelman, 2006). Hence, more extreme posterior samples, which a mean is relatively sensitive to, can only be positive. The mode, on the contrary, is less affected by extreme values, which could explain at least the absence of positive bias in the MAP variance estimates. Also, ML estimates of variances suffer from negative bias in finite samples if the corresponding means are unknown (Bishop, 2006). In addition, the employed inverse Gamma priors have a high concentration of mass near zero and are known to potentially drag smaller variances towards zero (Gelman, 2006).

I should note that the above elaborations, especially on bias patterns within conditions, remain in the speculative realm and require further support, for instance via sensitivity analyses investigating the effects of variations of prior distributions empirically. For the models at hand, the identification of *sources* of bias is further complicated by the fact that, although they are independent random variables a priori, model parameters become related a posteriori conditional on the data, potentially leading to compensatory biases.

Table 4.2 shows the performance of the univariate Gaussian AR model fitted as the true model (Section A), and fitted as a misspecified model to data generated from the PAR model with different average conditional location parameters (Sections B to E). The results in Section E are produced by the Gaussian AR model including the transformed latent process error. Sample size is kept constant across conditions (i.e., $N = 50, T = 30$).

For the Gaussian AR model as the true model (Section A), only a small number of solutions (i.e., less than 2 %) had to be excluded due to critical $\hat{R}$ values, the remaining solutions are based on sufficient numbers of effective samples, and all samplers adapted. The $\hat{R}$ estimates globally converge towards one with increasing chain length. Traceplots appear largely appropriate, but sometimes feature chains that show bad mixing behavior in that they get stuck in a specific, non-central posterior location for short sampling periods. For the misspecified Gaussian models without log link (Sections B to D), 3 to 9 % of the solutions were non-convergent. Also, traceplots contain more instantiations of bad mixing behavior sometimes even leading to bi-modal marginal posterior distributions, the more skewed the generated data. All these convergence issues resolve when the log link is included into the misspecified Gaussian AR model (Section E).

Table 4.2. Performance of the AR model fitted to data generated from the AR and the PAR model

Parameter		EAP			MAP			BCI 95%			Convergence	
Name	True	Bias	Bias%	RMSE	Bias	Bias%	RMSE	Cover	Width	Power	Rhat	Neff
A: AR model is true model												
κ	0.00	0.00	-	0.11	0.00	-	0.11	95.20	0.47	-	1.01	1815.52
β	0.60	-0.05*	-7.83*	0.10	-0.04*	-7.02*	0.11	90.20	0.31	100.00	1.01	816.41
ψ	0.40	0.10*	23.8*	0.16	0.06*	14.26*	0.17	90.20	0.50	100.00	1.02	878.95
θ	0.50	-0.07*	-14.91*	0.14	-0.05*	-10.24*	0.15	92.40	0.43	100.00	1.02	938.34
ϕ	0.50	0.20*	39.41*	0.36	0.06*	12.76*	0.28	90.60	1.13	100.00	1.01	1022.96
B: AR model is misspecified, data are highly skewed (i.e., $\kappa = -.7$)												
κ	-0.70	1.76*	250.73*	1.80	1.73*	246.82*	1.79	0.00	1.16	0.00	1.02	682.15
β	0.60	-0.14*	-23.45*	0.24	-0.16*	-26.59*	0.30	72.40	0.51	96.00	1.03	481.63
ψ	0.40	0.73*	182.43*	1.16	0.61*	152.06*	1.27	68.60	1.78	100.00	1.05	478.14
θ	0.00	1.44*	-	1.57	1.49*	-	1.74	0.00	1.67	-	1.05	619.87
ϕ	0.50	0.45*	89.93*	1.50	0.24*	48.56*	1.29	77.00	1.76	100.00	1.03	652.60
C: AR model is misspecified, data are moderately skewed (i.e., $\kappa = 0$)												
κ	0.00	1.88*	-	2	1.82*	-	1.97	0.00	1.72	-	1.02	745.69
β	0.60	-0.08*	-13.95*	0.19	-0.09*	-14.81*	0.22	71.00	0.37	100.00	1.03	527.85
ψ	0.40	2.54*	633.88*	3.10	2.41*	602.94*	3.29	6.20	3.41	100.00	1.04	587.70
θ	0.00	3.04*	-	3.32	3.07*	-	3.54	0.00	3.04	-	1.04	698.16
ϕ	0.50	2.27*	453.73*	3.83	1.80*	359.86*	3.49	45.80	4.46	100.00	1.02	729.21
D: AR model is misspecified, data are weakly skewed (i.e., $\kappa = .7$)												
κ	0.70	2.82*	403.47*	3.02	2.73*	389.35*	2.95	2.00	2.76	100.00	1.02	734.09
β	0.60	-0.05*	-8.13*	0.14	-0.05*	-8.22*	0.16	69 .60	0.28	100.00	1.03	456.17
ψ	0.40	9.85*	2462.50*	11.48	9.62*	2406.18*	11.62	0.00	8.03	100.00	1.03	568.10
θ	0.00	6.18*	-	7.02	6.22*	-	7.39	0.00	6.62	-	1.04	631.45
ϕ	0.50	8.72*	1743.26*	11.24	7.31*	1462.20*	9.89	4.40	13.33	100.00	1.02	701.90
E: AR model is misspecified, but includes the transformed process error, data are moderately skewed (i.e., $\kappa = 0$)												
κ	0.00	0.80*	-	0.80	0.80*	-	0.81	0.00	0.46	-	1.01	820.81
β	0.60	-0.47*	-78.88*	0.48	-0.47*	-78.94*	0.48	0.00	0.12	96.40	1.00	2952.95
ψ	0.40	-0.04*	-8.82*	0.05	-0.04*	-9.70*	0.05	74.20	0.12	100.00	1.00	3323.47
θ	0.00	0.88*	-	0.89	0.87*	-	0.88	0.00	0.39	-	1.00	3845.04
ϕ	0.50	0.06*	12.94*	0.14	0.02*	3.94*	0.12	93.40	0.49	100.00	1.00	3938.30

Note. True = true value, used for data generation; Cover = coverage in %; power in %; Rhat = $\hat{R}$; Neff = number of effective samples; absolute values below or equal to .005 are rounded to 0; * p < (.05); performance for θ is calculated as if the true value was 0. The number of replications is 500, and $N = 50$, $T = 30$ per condition A - E.

Interestingly, the Gaussian AR model performs worse as the true model (Section A) than the PAR model performs as the true model (cf. Table 4.1, Section A). That is, the MAP and especially the EAP estimates of all variance components are biased substantially, and to a larger extent than under the PAR model. Note, however, that these biases occur given relatively small sample sizes and can be expected to decrease with increasing amounts of data[8]. Also, BCIs for all parameters perform sufficiently well.

In case of the ordinary Gaussian AR model fitted to data generated from different variants of the PAR model (Sections B to D), it is not very surprising that most of the parameters are not recovered well as this model structure is severely misspecified. I would therefore like to draw attention to the AR effect β, which is the only parameter playing a comparable role in the PAR and the AR model. Also, it is often of substantive interest (e.g., De Haan-Rietdijk, Gottman, et al., 2016). This parameter is underestimated substantially, and to a degree that exceeds the biases under the AR and PAR models fitted as the true models in the same sample size condition. Accordingly, the corresponding coverage rate of the BCI is too low.

The skewness of the generating data modulates model performance, as expected. Highly skewed data (i.e., $\kappa = -.7$, Section B) lead to a larger negative AR effect-bias than moderately skewed data (i.e., $\kappa = 0$, Section C), than weakly skewed data (i.e., $\kappa = .7$, Section D). Interestingly, BCI coverage rates do not follow this clear trend – or may even show a reversed trend. Here, interval width, which relates positively to skewness, seems to play a compensating role.

One might now attribute these effects of misspecification to the missing log link rather than to the missing Poisson distribution. However, as Section E demonstrates, including the log link does worsen the situation for the AR effect. The size of bias in the point estimates increases drastically and coverage of the BCI drops to zero percent. What is improved is the recovery of the IA latent residual process variance (cf. parameter ψ) and the IE variance component (cf. parameter ϕ).

These simulations strongly support the suggestion to use the Poisson distribution in the measurement model when modeling count TS, especially if data are skewed. For more symmetrical event count distributions, it may well be that the Gaussian model works as a good approximation, at least with respect to the AR effect.

Table 4.3 displays the results for the hPVAR model under two sample size conditions with varying number of time points (i.e., $N = 50, T = (30{,}50)$). Per condition, I observe around 16 % non-converging solutions as measured by $\hat{R}$ and an increment of 1 – 4 % due to insufficient numbers of posterior samples. Whereas the $\hat{R}$ series show global convergence towards one with increasing chain length, the traceplots reveal chains that remain highly auto-correlated after

[8] And they in fact do, as preliminary simulations show.

thinning, and thus move relatively slowly and do not mix well, especially for the AR and CR effects and the conditional location parameters, and to a minor extent also for the latent process error variances.

Performance in terms of BCI coverage rates seems acceptable for all parameters but the event rate residual process variance ψ_{EE} and the corresponding covariance ψ_{EA} in both conditions, and the average intercept of the event rate process κ_E in the condition with least data. Biases of the MAP estimates largely mirror these patterns, EAP estimates additionally suffer from substantial miss-estimation of the inter-individual variance components ϕ_{EE}, ϕ_{EA}, and ϕ_{AA}. These asymmetries in performance (i.e., parameters associated with the event rate process seem to be less well recovered) may be attributable to asymmetries in model structure, which involve the parameter values chosen to generate the data, but also the process-specific measurement models.

Note that the BCIs in the corresponding Gaussian VAR model perform slightly better under conditions being partly comparable (i.e., asymmetries in the data-generating parameter values are invariant across models), as displayed in Table 4.4, Section A. For this model, however, I also observe an increased non-convergence rate of 32 % due to $\hat{R}$ and of incremental 5 % due to effective posterior sample size.

For both the VAR and the hPVAR model, the number of efficient samples is quite low in comparison to the simulation results from the univariate models, at least as far as the correctly specified models are concerned. Under the hPVAR model, this effect is most pronounced for the intra-individual process parameters and reflects the earlier traceplot behavior-observations. Under the VAR model (true and misspecified), traceplots reveal chains that are not as strongly auto-correlated, but show transient periods of non-stationary behavior.

Finally, Section B of Table 4.4 shows the performance of the Gaussian VAR model fitted to data generated from the hPVAR model. These results are to be interpreted with caution, because they are calculated from only 18 out of 500 replications. Accordingly, there are extensive convergence problems (i.e., a non-convergence rate of 92 % as measured by $\hat{R}$, and incremental 5 % due to insufficient number of effective samples). Section B of Table 4.4 however still conveys that the Gaussian VAR model provides an inappropriate description if the data are generated under the hPVAR model.

Table 4.3. Performance of the hPVAR model fitted to data generated from the hPVAR model

Parameter		EAP			MAP			BCI 95%			Convergence	
Name	True	Bias	Bias%	RMSE	Bias	Bias%	RMSE	Cover	Width	Power	Rhat	Neff
A: $N = 50, T = 30$												
κ_E	0.00	-0.21*	-	0.35	-0.18*	-	0.33	85.60	1.04	-	1.06	119.42
κ_A	0.00	-0.07*	-	0.19	-0.06*	-	0.19	91.60	0.67	-	1.06	145.73
β_{EE}	0.60	0.01*	2.31*	0.07	0.02*	3.59*	0.07	92.00	0.23	100.00	1.07	93.84
β_{AE}	0.30	0.00	-0.84	0.03	0.00*	-1.39*	0.03	95.60	0.11	100.00	1.04	198.08
β_{EA}	0.20	0.00	1.16	0.04	0.00	-0.53	0.04	90.80	0.16	100.00	1.06	118.04
β_{AA}	0.50	0.01*	1.14*	0.03	0.01*	1.69*	0.03	94.80	0.12	100.00	1.03	184.83
ψ_{EE}	0.40	0.12*	30.29*	0.18	0.08*	19.51*	0.14	86.40	0.52	100.00	1.05	183.59
ψ_{EA}	0.10	0.03*	34.61*	0.06	0.03*	27.76*	0.06	88.00	0.21	76.00	1.01	1205.30
ψ_{AA}	0.40	0.04*	9.01*	0.07	0.02*	5.33*	0.07	92.80	0.26	100.00	1.02	356.92
θ_A	0.50	0.02*	3.12*	0.04	0.01*	2.52*	0.04	93.20	0.15	100.00	1.00	1644.86
ϕ_{EE}	0.50	0.08*	15.56*	0.18	0.01	1.49	0.13	96.00	0.63	100.00	1.02	789.86
ϕ_{EA}	0.10	-0.03*	-34.82*	0.11	-0.03*	-26.13*	0.09	94.80	0.42	12.40	1.01	828.05
ϕ_{AA}	0.50	0.06*	11.15*	0.15	-0.01	-1.24	0.12	95.60	0.57	100.00	1.01	1097.16
B: $N = 50, T = 50$												
κ_E	0.00	-0.11*	-	0.24	-0.09*	-	0.24	93.20	0.80	-	1.07	118.52
κ_A	0.00	-0.01	-	0.14	0.00	-	0.14	96.00	0.56	-	1.08	123.39
β_{EE}	0.60	0.01*	1.17*	0.05	0.01*	1.79*	0.05	90.00	0.18	100.00	1.06	104.97
β_{AE}	0.30	0.00*	-1.45*	0.02	-0.01*	-1.74*	0.02	92.40	0.08	100.00	1.04	202.40
β_{EA}	0.20	0.00	0.58	0.04	0.00	-0.62	0.04	91.20	0.12	100.00	1.05	128.05
β_{AA}	0.50	0.00*	0.97*	0.03	0.01*	1.35*	0.03	93.60	0.09	100.00	1.03	192.53
ψ_{EE}	0.40	0.06*	14.61*	0.11	0.04*	9.08*	0.09	88.80	0.34	100.00	1.04	219.81
ψ_{EA}	0.10	0.02*	21.05*	0.04	0.02*	18.4*	0.04	88.40	0.14	92.80	1.01	1452.88
ψ_{AA}	0.40	0.01*	3.51*	0.05	0.01*	1.55*	0.05	94.00	0.19	100.00	1.02	397.76
θ_A	0.50	0.01*	1.96*	0.03	0.01*	1.54*	0.03	92.80	0.12	100.00	1.00	1674.82
ϕ_{EE}	0.50	0.06*	11.03*	0.14	0.00	0.52	0.11	95.60	0.55	100.00	1.01	1082.63
ϕ_{EA}	0.10	-0.02*	-17.93*	0.08	-0.02*	-18.23*	0.07	96.80	0.36	16.00	1.01	1222.14
ϕ_{AA}	0.50	0.02*	3.53*	0.11	-0.03*	-6.26*	0.10	97.60	0.49	100.00	1.01	1556.09

Note. True = true value, used for data generation; Cover = coverage in %; power in %; Rhat = $\hat{R}$; Neff = number of effective samples; absolute values below or equal to .005 are rounded to 0; * p < (.05). The number of replications is 250 per condition A and B.

Table 4.4. Performance of the VAR model fitted to data generated from the VAR and the hPVAR model

Parameter		EAP			MAP			BCI 95%			Convergence	
Name	True	Bias	Bias%	RMSE	Bias	Bias %	RMSE	Cover	Width	Power	Rhat	Neff
A: VAR model is true												
κ_E	0.00	0.01*	-	0.12	0.01	-	0.11	93.60	0.45	-	1.01	870.62
κ_A	0.00	0.00	-	0.11	0.00	-	0.10	94.40	0.44	-	1.01	1142.28
β_{EE}	0.60	-0.02*	-3.47*	0.10	0.00	0.19	0.12	96.00	0.40	100.00	1.08	129.81
β_{AE}	0.30	0.01*	4.43*	0.09	-0.01	-2.77	0.11	92.80	0.36	100.00	1.07	159.11
β_{EA}	0.20	0.02*	11.19*	0.09	-0.01	-3.34	0.10	96.00	0.35	93.60	1.08	137.48
β_{AA}	0.50	-0.01*	-2.94*	0.10	0.00	-0.90	0.14	92.40	0.40	100.00	1.07	158.03
ψ_{EE}	0.40	0.08*	19.67*	0.15	0.03*	7.81*	0.17	91.20	0.49	100.00	1.07	183.83
ψ_{EA}	0.10	0.01*	12.49*	0.03	0.01*	11.59*	0.03	93.60	0.10	100.00	1.00	2084.53
ψ_{AA}	0.40	0.09*	21.34*	0.17	0.04*	9.86*	0.20	94.40	0.54	100.00	1.06	232.05
θ_E	0.50	-0.06*	-11.11*	0.13	-0.03*	-5.28*	0.16	93.20	0.44	100.00	1.07	201.92
θ_A	0.50	-0.07*	-13.76*	0.15	-0.04*	-8.30*	0.20	95.20	0.50	100.00	1.06	233.99
ϕ_{EE}	0.50	0.10*	20.81*	0.27	-0.03*	-6.66*	0.23	97.20	1.05	100.00	1.05	307.23
ϕ_{EA}	0.10	0.00	0.90	0.14	-0.04*	-37.32*	0.12	95.60	0.58	7.60	1.01	863.31
ϕ_{AA}	0.50	0.10*	19.29*	0.24	-0.03	-5.16	0.21	97.20	0.97	100.00	1.04	356.34
B: VAR model is misspecified												
κ_E	0.00	1.43*	-	1.49	1.35*	-	1.42	0.00	1.76	-	1.03	393.43
κ_A	0.00	1.69*	-	1.75	1.64*	-	1.70	0.00	1.59	-	1.04	303.66
β_{EE}	0.60	0.02	3.38	0.13	0.03	4.73	0.17	88.89	0.38	100.00	1.10	129.36
β_{AE}	0.30	-0.10*	-34.60*	0.13	-0.12*	-41.55*	0.15	66.67	0.26	94.44	1.12	97.02
β_{EA}	0.20	-0.01	-5.15	0.12	-0.01	-3.13	0.14	88.89	0.29	72.22	1.09	153.14
β_{AA}	0.50	0.09*	17.09*	0.13	0.10*	19.89*	0.14	66.67	0.27	100.00	1.10	81.60
ψ_{EE}	0.40	2.15*	538.29*	3.02	1.57*	393.57*	2.05	0.00	3.50	100.00	1.09	198.25
ψ_{EA}	0.10	0.25*	254.66*	0.34	0.24*	244.77*	0.32	66.67	0.55	66.67	1.01	932.30
ψ_{AA}	0.40	1.41*	352.12*	1.60	1.31*	327.11*	1.46	0.00	1.20	100.00	1.04	222.35
θ_E	0.50	6.29*	1258.24*	6.46	6.62*	1324.56*	6.72	16.67	3.49	100.00	1.07	338.90
θ_A	0.00	0.78*	-	0.85	0.84*	-	0.94	0.00	0.97	-	1.05	183.88
ϕ_{EE}	0.50	0.86*	172.61*	1.56	0.49	97.57	1.22	83.33	2.69	100.00	1.05	275.26
ϕ_{EA}	0.10	0.39*	389.75*	0.60	0.24*	239.01*	0.44	83.33	1.74	16.67	1.04	398.58
ϕ_{AA}	0.50	1.47*	293.62*	2.21	1.13*	225.74*	1.78	38.89	2.94	100.00	1.04	250.55

Note. True = true value, used for data generation; Cover = coverage in %; power in %; Rhat = $\hat{R}$; Neff = number of effective samples; absolute values below or equal to .005 are rounded to 0; * p < (.05); performance for θ_E is calculated as if the true value was 0. The number of replications is 250, and $N = 50$, $T = 30$ per condition A and B.

4.5 Application

4.5.1 Data selection

For the present analyses, as for the application presented in Chapter 3, Section 3.4 (cf. Adolf et al., 2017), I rely on data gathered in context of the COGITO study. The design, participants and measures of this intense longitudinal study are described in more detail in Chapter 3, Section 3.4.1 (see also Schmiedek et al., 2010). I again analyze negative affect as assessed by the Positive and Negative Affect Schedule (Watson et al., 1988), and daily events assessed within seven different event domains (e.g., work, health, leisure, finances) based on the Daily Inventory of Stressful Experiences (Almeida, Wethington, & Kessler, 2002) and other event questionnaires (Zautra, Affleck, & Tennen, 1994). For negative affect, I again rely on the average score across a selection of the five most variable negative affect items (i.e., "distressed", "upset", "irritable", "nervous", and "jittery"), which were answered on an 8-point scale. For events, I rely on the number of events across domains that are reported as negative and as having happened before a given measurement occasion or as being ongoing at a given measurement occasion.

To keep the amount of data in a moderate range, and to further maximize variability in the data, I do not use the 87 to 109 sessions from all 204 participants. Instead, I confine the present illustration to a subset of 50 participants and 50 measurement occasions on consecutive days. As shown in the simulation study, the PAR model performs very well and the hPVAR model still acceptable for these sample sizes. The data selection procedure was as follows: I first again up-sampled the data to the resolution of days, inserting missing values at each non-measured day (see Chapter 3, Section 3.4.3). Per participant, I then searched the data set for the 50 occasion-period with minimal numbers of missing values. Note that since I assume stationary processes here, it should not matter which period out of the total study period I analyze, or whether I select individual-specific periods. Only the first five occasions are always excluded because of potential reactivity effects during this very initial study period. Based on these within-person selections of occasions, I then chose those 50 participants who display maximal variability on, and minimal within-occasion correlations between negative affect and the number of negative events.

The decision to select data based on maximal variability and minimal within-occasion correlations between the two variables of interest was taken to facilitate empirical model identification, specifically, the identification of cross-lagged relationships[9]. Of course, selecting a sample based on properties of the variables to be modelled, especially on co-variances, might lead to results that are no longer generalizable to the population the original COGITO sample was drawn from. Whereas I consider this relatively unproblematic for the present illustration, it might be an issue in substantive research.

4.5.2 Model building and fitting

Given the selected subset of data, I start with fitting univariate models to the event and affect TS separately. In case of the event data, I fit and compare the PAR and the Gaussian AR model. In case of the negative affect data, I fit and compare Gaussian AR models without and with a transformed latent process error (cf. Chapter 4, Section 4.4.1), as the latter model structure functions as a building block in the bivariate hybrid model. The hPVAR model is fitted subsequently, and compared to a Gaussian VAR model.

The general estimation procedure is implemented as described in Chapter 4, Section 0, including the setup of the prior distributions presented in Chapter 4, Section 4.4.2. For all models, I increase the number of iterations during the simulation of the posterior, that is, I let JAGS draw 550,000 samples, of which the first 50,000 are discarded, and thin by a factor of 500.

4.5.3 Model evaluation and comparisons

I evaluate model fit via posterior predictive checks. Gelman and colleagues (Gelman et al., 2004) define posterior predictive assessments of model fit as any comparison of observed (or new) data with model-implied data generated from the posterior predictive distribution. The posterior predictive distribution is the likelihood function weighted by the posterior probabilities of and marginalized over the model parameters. When relying on simulation-based estimation, the posterior predictive distribution can be approximated by mixing the likelihood function according to the posterior samples drawn. Posterior predictive checks may be conducted at the level of the raw data, or with respect to statistics derived from the data.

In the present context, I use posterior predictive checking less for rigorous model testing, but in a more heuristic and explorative manner by comparing posterior predictive and actual

[9] The bivariate model did not converge in previous analyses with data selected only with respect to the number of missing occassions and observed variability.

trajectories at the individual level and speculating about potential sources of (mis-)fit. To enable this, I let JAGS sample the latent process states per time point and use the EAP estimates of these – and the EAP estimate of the Gaussian measurement error variance, where necessary – to obtain predictive quantiles for the data according to the measurement models in Equations (4.6), (4.10) and (4.11). Specifically, I calculate the 15.87 % and 84.13 % quantiles – in terms of the standard normal distribution the quantiles one standard deviation below and above the mean – to determine the interval within which about two thirds of all values are expected to lie. I do not marginalize over the posterior, neither for the latent process states, nor for the involved parameter estimates. Rather than incorporating the uncertainty of the posterior, I condition on typical posteriors values, which renders this a relatively conservative model check. I plot the results and evaluate them by visual inspection.

Model comparisons between alternative models are conducted using the expected deviance as a measure of model fit, penalized by the effective number of parameters as a measure of model complexity. This is known as the deviance information criterion (DIC; Plummer, 2008; Spiegelhalter, Best, Carlin, & van der Linde, 2002, 2014). As the DIC has been shown to be overly liberal with respect to complex models in certain situations, I additionally report the expected deviance combined with an alternative penalty term as proposed by Plummer (2008). Lower values indicate better fit for both indices. In the context of the individual-level posterior predictive checks, I also report *individual contributions* to the DIC and to the alternatively penalized deviance. These local fit measures are calculated per person as the sum of the individual contribution to the expected deviance and the individual contribution to the penalty term, relative to the number of non-missing occasions a person provides. The expected deviance and the two penalty terms can be obtained per datum via simulation-based estimation in JAGS.

4.5.4 Results with discussion

For all models, plots of the marginal posterior distributions and traceplots per parameter of interest are included in Appendix H. Except for the hPVAR model, traceplots consist of chains that show largely acceptable behavior, sometimes displaying disruptions from mixed, stationary patterns that do not last very long, but occur across chains and reveal how model

Table 4.5.Results from univariate models fitted to COGITO event data

Parameter	EAP	MAP	BCI 95%		Convergece		Fit		
			LowerB	UpperB	Rhat	Neff	ExpDev	DIC	PenDev
A: PAR model									
κ	-4.00	-3.86	-6.03	-2.32	1.00	382.51	3197.26	**3286.34**	**3386.35**
β	0.90	0.92	0.77	0.97	1.05	370.47			
ψ	1.64	1.23	0.14	3.97	1.00	411.10			
ϕ	0.45	0.42	0.27	0.72	1.00	3881.24			
B: AR model									
κ	0.11	0.09	0.05	0.20	1.00	2031.97	3376.06	3673.42	4082.91
β	0.83	0.85	0.69	0.92	1.00	1920.62			
ψ	0.04	0.03	0.02	0.07	1.00	2072.80			
θ	0.38	0.38	0.34	0.42	1.00	3542.54			
ϕ	0.01	0.00	0.00	0.02	1.00	2116.87			

Note. LowerB = lower bound; UpperB = upper bound; Rhat = $\hat{R}$; Neff = number of effective samples; ExpDev = expected deviance; PenDev = expected deviance with alternative penalty term; absolute values below or equal to .005 are rounded to 0; $N = 50$, $T = 50$; per fit measure, the best fitting model is printed in bold.

parameters depend on each other (e.g., for the PAR model). Samplers always adapted during the initial sampling phase.

Table 4.5 shows solutions obtained under the Poisson and Gaussian AR models fitted to the event TS. Both solutions yield high AR effects and thus indicate high temporal stability in the rate of negative events for the average individual. The *amount* and *nature* of variability the solutions imply, however, seems to constitute a major difference. That is, under the PAR model (Section A) there is IA latent process error variability (cf. parameter ψ), that is, for the average person the model suggests perturbations to the event rate process, which then take a long time to diminish. Also, there is IE variability in conditional locations (cf. parameter ϕ). Under the Gaussian AR model (Section B) the amount of variability from these two sources is estimated close to zero. Instead, there is some measurement error variance accounting for fluctuations in event counts that are not time-structured (cf. parameter θ). Note, however, that the above referenced parameters are not directly comparable across models due to differences in model-structure and implied distributional shapes. If I simulate data for an average "model-implied person" using the obtained EAP estimates, I get a highly skewed distribution peaking near zero also for the PAR model. According to both fit measures, the DIC and the expected deviance with alternative penalty term, the PAR model fits the event data better than the Gaussian AR model.

In Figure 4.4, I plot a selection of posterior predictive and actual trajectories at the individual level. The selection includes the six individuals with the lowest and highest contributions to the DIC for the PAR model in Panel A. Alongside with these, I display the corresponding Gaussian AR model solutions in Panel B. So, across panels, each row represents an individual case and solutions from the two competing models. Note that for the ordinary AR model solutions, the individual DIC values are no longer in strict ascending order. In fact, there is basically no rank order correlation if individuals are sorted according to their contributions to model fit between the PAR and the AR model (i.e., $\tau = -.09$ for the DIC, and $\tau = .01$ for the alternatively penalized deviance)[10]. Still, it is evident from the upper half of the figure that both models provide better fitting solutions to cases with rather stable trajectories over time.

The PAR model thereby seems to do better than the AR model (i.e., fit values are lower) in describing skewed distributions, produced by individuals whose event reports are at floor most of the time, and just show occasional deviations, necessarily in the positive direction. The AR model, on the other hand, fits those three cases better for whom the PAR model shows the largest misfit. Here, the comparably bad performance of the PAR model might be due to the fact, that the model implies fluctuations that scale in magnitude with mean level. Remember that this is the case for the transformed process error at the latent level given the present model setup, and for the measurement error at the manifest level, as the variance of a Poisson distribution is equal to its mean. Accordingly, the three cases presented in the lower half of Figure 4.4 display relatively little variation, but at elevated mean levels.

[10] Sorting individuals *within* the PAR model according to their individual contributions to the DIC and according to the alternatively penalized deviance produces almost exactly the same rank order ($\tau = .98$). Using the individual contributions to the alternatively penalized deviance thus produces a similar figure.

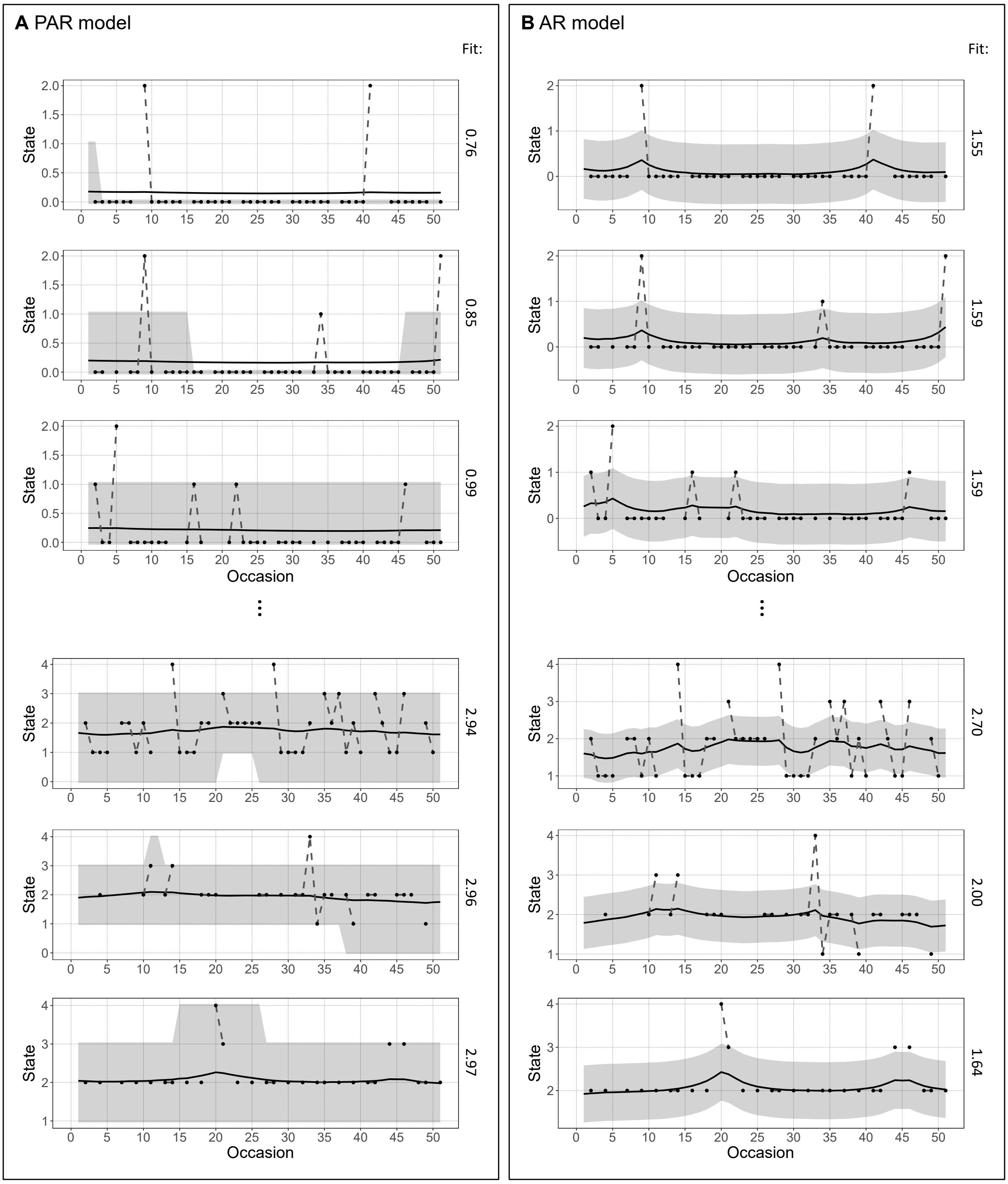


Figure 4.4. Posterior predictive trajectories under the PAR and AR model, plotted against the analyzed COGITO event data. Dots connected by dashed lines signify observed data. Solid black lines show model-implied trajectories based on the EAP estimates of the latent process states, that is, event rates, per time point. Light grey areas cover the interval within which approximately 68 % of the observations, that is, event counts, are expected to fall according to the respective measurement models. Each row contains a single case across both Panels A and B. Cases are sorted in ascending order according to their individual contributions to the DIC under the PAR model. Individual DIC values are printed right to each trajectory plot for both models.

Table 4.6. Results from univariate models fitted to COGITO affect data

Parameter	EAP	MAP	BCI 95%		Convergece		Fit		
			LowerB	UpperB	Rhat	Neff	ExpDev	DIC	PenDev
A: AR model log link									
κ	-0.30	-0.31	-0.55	-0.05	1.00	1448.57	2076.49	**2999.64**	6974.59
β	0.30	0.31	0.22	0.39	1.00	2400.70			
ψ	0.27	0.27	0.22	0.33	1.00	2691.82			
θ	0.19	0.19	0.15	0.22	1.00	3682.67			
ϕ	0.53	0.49	0.35	0.81	1.00	3887.34			
B: AR model									
κ	0.12	0.10	0.05	0.24	1.00	486.64	3624.31	3913.13	**4310.44**
β	0.92	0.93	0.84	0.96	1.00	514.39			
ψ	0.03	0.03	0.02	0.06	1.00	703.58			
θ	0.44	0.43	0.40	0.48	1.00	2585.92			
ϕ	0.01	0.00	0.00	0.02	1.01	648.48			

Note. LowerB = lower bound; UpperB = upper bound; Rhat = $\hat{R}$; Neff = number of effective samples; ExpDev = expected deviance; PenDev = expected deviance with alternative penalty term; absolute values below or equal to .005 are rounded to 0; $N = 50$, $T = 50$; per fit measure, the best fitting model is printed in bold.

Table 4.6 shows the solutions under the Gaussian AR model with and without log link fitted to the negative affect TS (i.e., high auto-stability, little variation at the latent level, some variation at the manifest level). While the ordinary AR model (Section B) leads to results that appear very similar to those obtained for the event TS, the AR model including the log link and thus featuring the transformed latent error process, draws a different picture (cf. Section A). Here, the AR effect is of small size and there is time-structured and time-unstructured variability within the average person (cf. parameters ψ and θ), and variability between persons (cf. parameter ϕ). A direct comparison of the models in terms of the parameters pertaining to the latent process (i.e., κ, ψ, and ϕ) is again limited, as the latent process models imply different distributional shapes. The DIC favors the AR model with log link whereas the alternative fit index penalizes this model much more and favors the ordinary AR model.

Figure 4.5 yields posterior predictive trajectories under both models plotted against the observed data. Displayed are again the six individual cases with the lowest and highest DIC contributions, this time under the AR model with log link (Panel A), and the corresponding ordinary AR model solutions (Panel B). According to the DIC, the AR model with log link seems to do well, and better than the ordinary AR model, in the presence of skewed distributions with both less and more fluctuations over time, as displayed in the upper and lower half, respectively.

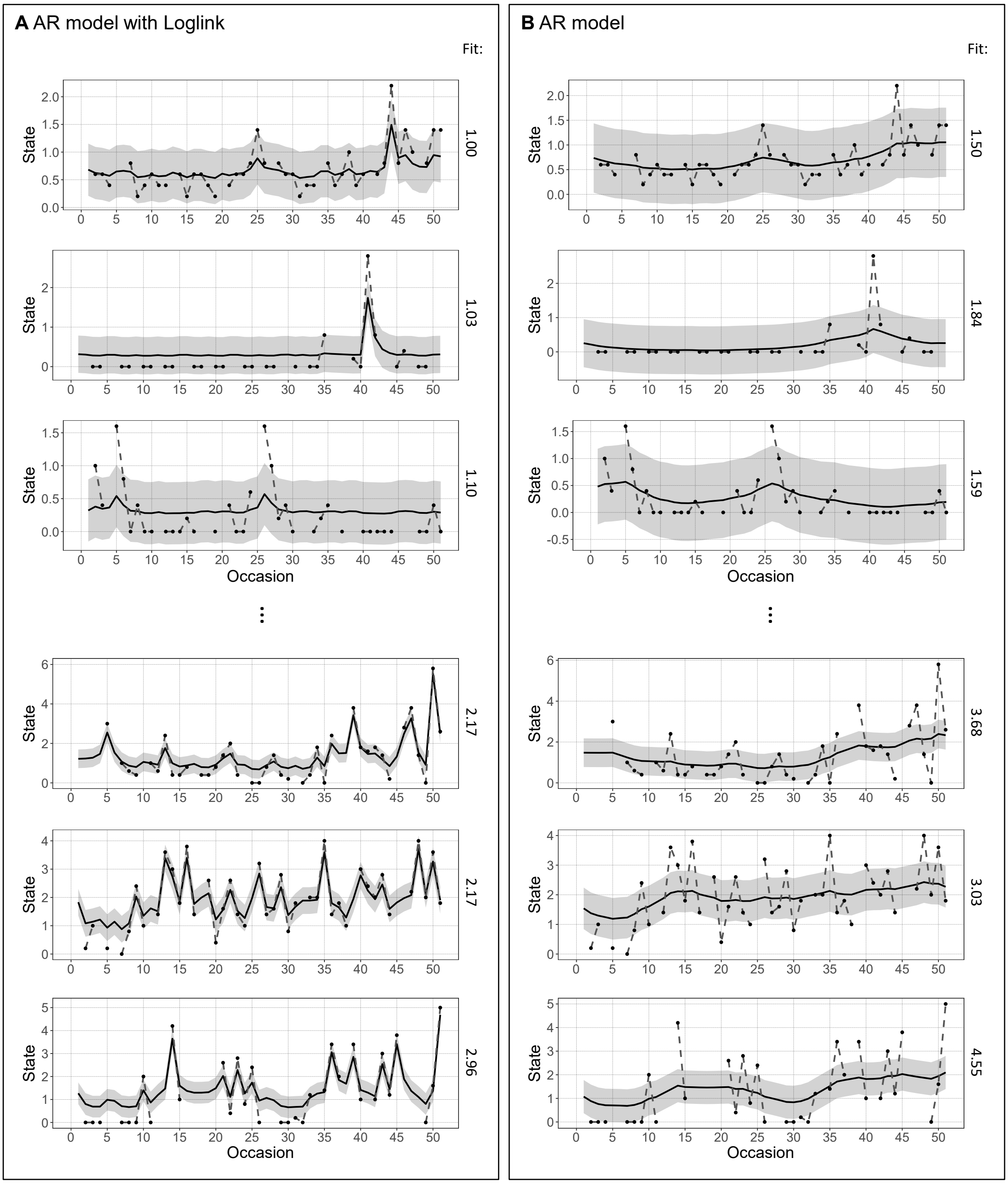


Figure 4.5. Posterior predictive trajectories under the AR model with and without log link, plotted against the analyzed COGITO affect data. Dots connected by dashed lines signify observed data. Solid black lines show model-implied trajectories based on the EAP estimates of the latent process states, that is, latent affect states, per time point. Light grey areas cover the interval within which approximately 68 % of the observations, that is, reported affect states, are expected to fall according to the respective measurement models. Each row contains a single case across both Panels A and B. Cases are sorted in ascending order according to their individual contributions to the DIC under the AR model with log link. Individual DIC values are printed right to each trajectory plot for both models.

Under the AR model with log link, individual contributions to the DIC have little implications for individual contributions to the alternatively penalized deviance ($\tau = .02$). Figure 4.6 therefore contrasts posterior predictive trajectories and observed data for both models based on the highest and lowest individual contributions to the alternatively penalized deviance. Whereas the upper halves of Figure 4.6 and Figure 4.5 appear to present similiar results, a comparison of the lower halves yields that the ordinary AR model may perform better for cases whose fluctuations follow rather symmetrical distributional shapes with mean levels farther away from zero. This would make sense, as such patterns should be less expected under the AR model featuring the transformed, lognormal process error.

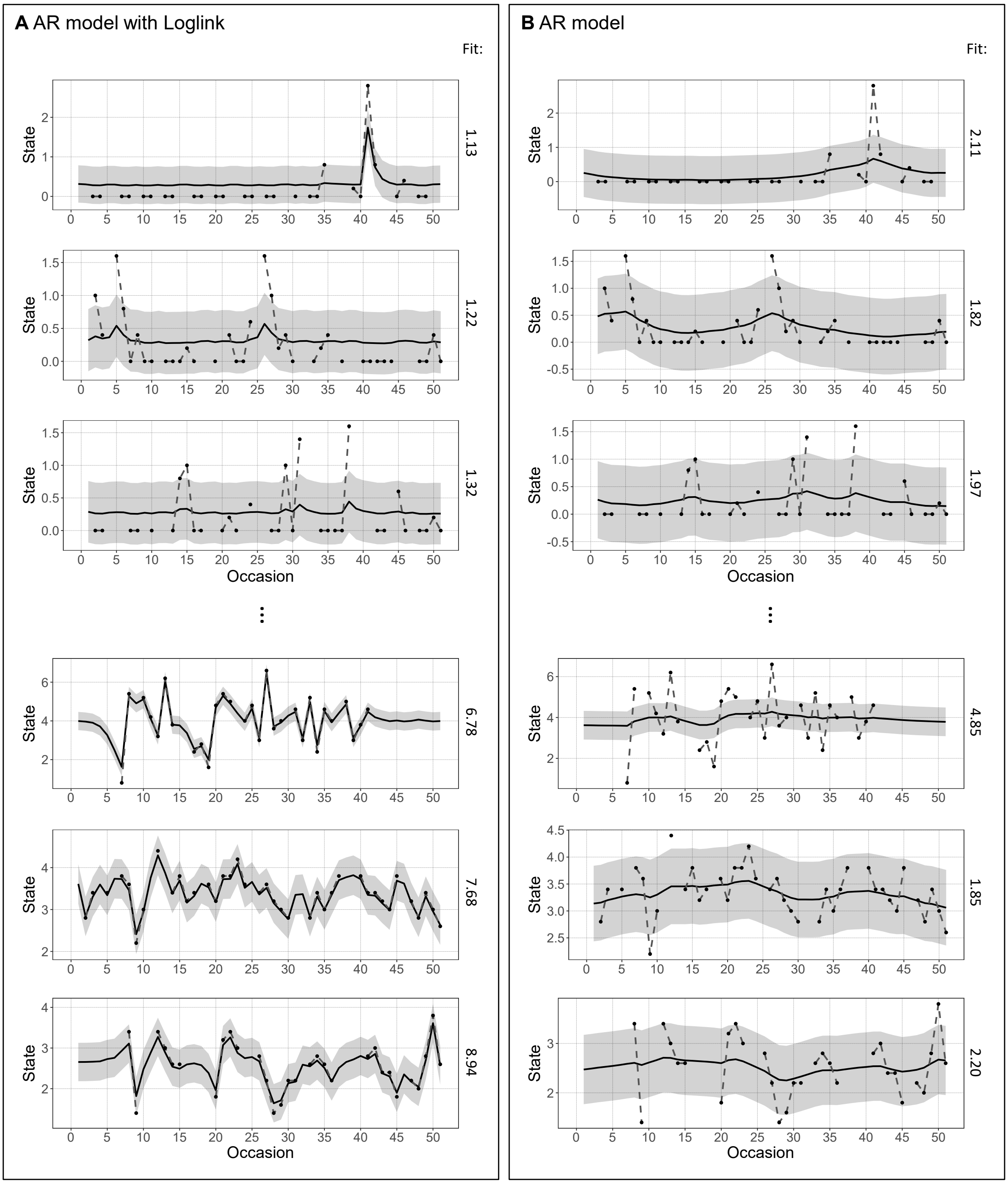


Figure 4.6. Posterior predictive trajectories under the AR model with and without log link, plotted against the analyzed COGITO affect data. Dots connected by dashed lines signify observed data. Solid black lines show model-implied trajectories based on the EAP estimates of the latent process states, that is, latent affect states, per time point. Light grey areas cover the interval within which approximately 68 % of the observations, that is, reported affect states, are expected to fall according to the respective measurement models. Each row contains a single case across both Panels A and B. Cases are sorted in ascending order according to their individual contributions to the alternatively penalized expected deviance under the AR model with log link. Individual contributions to the alternatively penalized expected deviance are printed right to each trajectory plot for both models.

Table 4.7 Results from bivariate models fitted to COGITO event and affect data

Parameter	EAP	MAP	BCI 95%		Convergece		Fit		
			LowerB	UpperB	Rhat	Neff	ExpDev	DIC	PenDev
A: hPVAR model									
κ_E	-0.66	-0.60	-0.87	-0.47	1.09	50.64	-1815.38	**3653.38**	**32339.55**
κ_A	0.48	0.52	0.32	0.64	1.85	55.35			
β_{EE}	0.23	0.18	0.08	0.41	1.02	45.49			
β_{AE}	-1.52	-1.46	-1.81	-1.20	1.64	56.37			
β_{EA}	-0.08	-0.07	-0.10	-0.06	1.05	182.29			
β_{AA}	0.37	0.34	0.27	0.45	1.30	51.61			
ψ_{EE}	0.05	0.05	0.02	0.09	1.79	84.83			
ψ_{EA}	0.07	0.07	0.04	0.10	1.34	89.42			
ψ_{AA}	0.14	0.14	0.11	0.18	1.41	94.73			
θ_A	0.01	0.00	0.00	0.02	1.04	508.03			
ϕ_{EE}	0.16	0.15	0.10	0.25	1.00	1054.40			
ϕ_{EA}	0.11	0.09	0.06	0.18	1.03	1630.01			
ϕ_{AA}	0.17	0.15	0.11	0.25	1.01	1548.63			
B: VAR model									
κ_E	0.56	0.57	0.37	0.75	1.00	1034.44	2502.99	23560.04	34437.27
κ_A	0.56	0.54	0.40	0.76	1.00	1506.30			
β_{EE}	0.22	0.18	0.12	0.38	1.01	658.10			
β_{AE}	-0.14	-0.13	-0.23	-0.07	1.00	1520.96			
β_{EA}	-0.05	-0.05	-0.16	0.06	1.01	942.56			
β_{AA}	0.68	0.69	0.56	0.77	1.00	1612.06			
ψ_{EE}	0.37	0.45	0.21	0.48	1.01	487.64			
ψ_{EA}	0.10	0.09	0.07	0.12	1.00	3254.35			
ψ_{AA}	0.13	0.12	0.09	0.18	1.00	1734.26			
θ_E	0.09	0.00	0.00	0.24	1.01	489.77			
θ_A	0.36	0.37	0.31	0.41	1.00	2382.50			
ϕ_{EE}	0.15	0.14	0.09	0.24	1.00	1939.03			
ϕ_{EA}	0.02	0.01	-0.03	0.06	1.00	2153.90			
ϕ_{AA}	0.10	0.08	0.05	0.18	1.00	1997.34			

Note. LowerB = lower bound; UpperB = upper bound; Rhat = $\hat{R}$; Neff = number of effective samples; ExpDev = expected deviance; PenDev = expected deviance with alternative penalty term; absolute values below or equal to .005 are rounded to 0; $N = 50$, $T = 50$; per fit measure, the best fitting model is printed in bold.

In Table 4.7, I eventually present the results from the hPVAR and the VAR model fitted to the event and affect TS, and in Figure 4.7 the accompanying posterior predictive trajectories.

Despite my data selection efforts, both alternative model solutions yield high within-time point relations of the two latent processes (cf. parameter ψ_{EA} relative to the variances ψ_{EE} and ψ_{AA}), conditional on the time-lagged effects of earlier process states. With respect to the latter, I am surprised to see – again for both models – negative cross-lagged effects in both directions.

The numerically large negative cross-lagged effect of earlier event rates on later latent affect in the hPVAR model is thereby likely attributable to the difference in the two latent processes' scales, which is also clearly visible in Figure 4.7. As the presented effects are unstandardized, their sizes are not interpretable in an absolute manner. Note that the high auto-regressive effects, that structured both latent processes in the univariate models, have substantially reduced, now that the models additionally allow for associations within time points and for cross-lagged effects. The hPVAR model is favored over the VAR model by both fit measures.

Concerning the dynamic effects under both models, I would rather have expected positive cross-lagged effects, as it seems plausible, that, for example, higher rates of negative events lead to higher levels of latent negative affect. Taking the time scale of the COGITO data into account (i.e., days), however, it is possible to come up with alternative post-hoc explanations that are not entirely implausible either. That is, given a sampling rate, which is low in comparison to typical experience sampling designs, it might well be that I here miss out on regulation processes that unfold at faster time scales – an intuition that also came up during the earlier model illustration using the COGITO data in Chapter 3, Section 3.4.6. That is, whereas higher negative rates may be associated with increases in latent negative affect initially, individuals might, as time goes by, take action and reduce negative affect successfully, eventually leading to the observed negative effects – although the positive AR effects do not match well with this story. In any case, unraveling the *time courses* of dynamic effects requires (parts of the) data to be sampled at higher rates, as well as appropriate analysis techniques, such as time-continuous dynamic modeling (Driver et al., 2017; Voelkle, 2017; Voelkle, Oud, Davidov, et al., 2012). Leaving theoretical speculations aside, it is of course also a possibility, that the here-concerned fist-order models are generally too simplistic and therefore misspecified. Unexpected effect patterns might at least be taken as a hint at this possibility.

Finally, I would like to caution against too strong and detailed interpretations of the associated parameter estimates also for another reason. As can be read off Table 4.7, convergence as measured by $\hat{R}$ is suboptimal for some of the model's parameters, and numbers of efficient samples from the marginal posterior distributions are low for all parameters pertaining to the within-person latent process part of the model. These issues are also mirrored in the corresponding traceplots in Appendix H, which, for the problematic parameters, contain highly auto-correlated, slowly moving chains that do not mix very well.

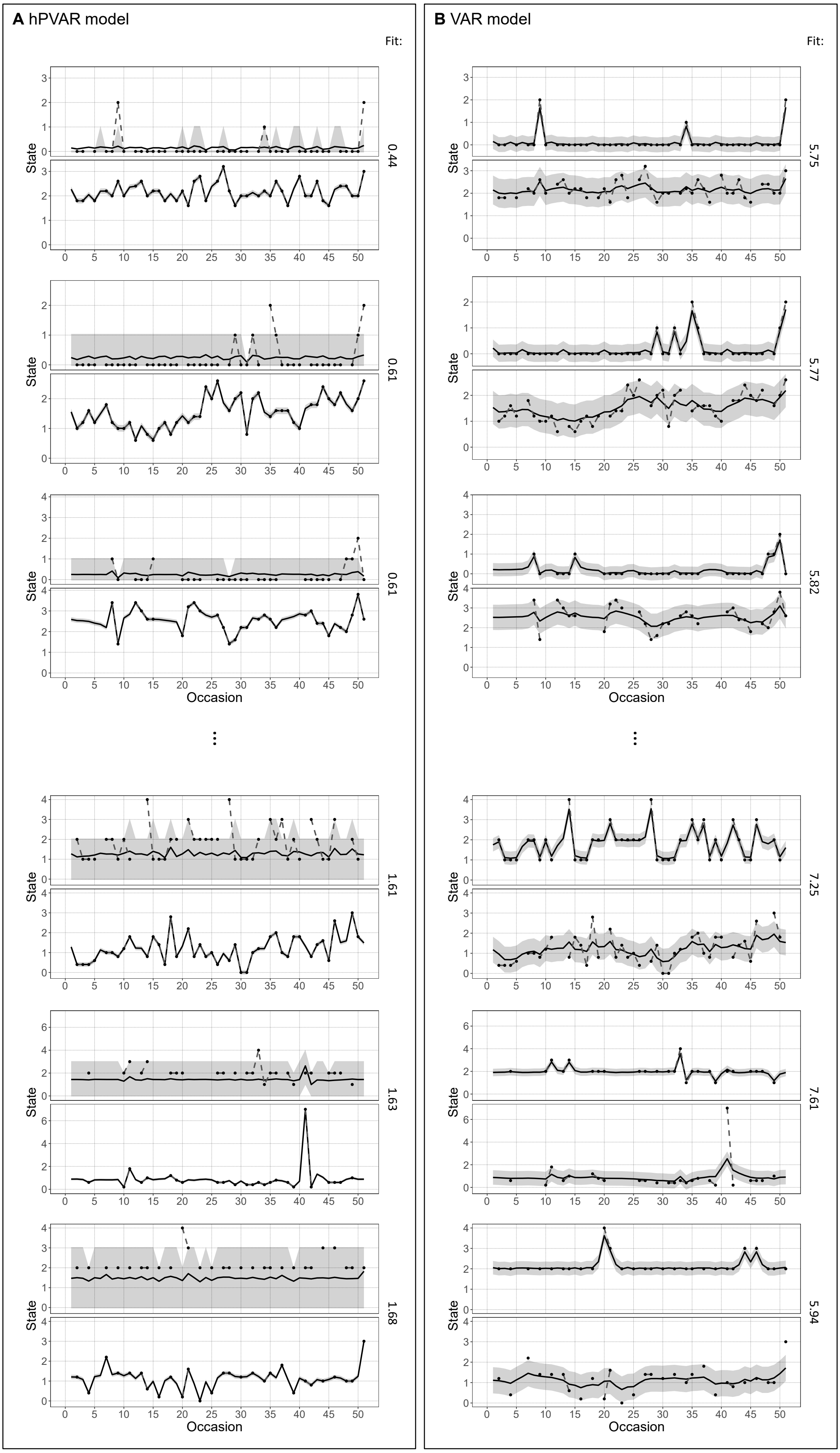

A hPVAR model
Fit:
0.44
0.61
0.61
1.61
1.63
1.68
State
Occasion
B VAR model
Fit:
5.75
5.77
5.82
7.25
7.61
5.94
State
Occasion

Figure 4.7. Posterior predictive trajectories under the hPVAR and the VAR model, plotted against the analyzed COGITO event and affect data. Dots connected by dashed lines signify observed data. Solid black lines show model-implied trajectories based on the EAP estimates of the latent process states per time point. Light grey areas cover the interval within which approximately 68 % of the observations are expected to fall according to the respective measurement models. Each row contains a single case across both Panels A and B. Cases are sorted in ascending order according to their individual contributions to the DIC under the hPVAR model. Individual DIC values are printed right to each trajectory plot for both models.

4.6 Discussion

Although contextual factors receive increasing attention in the affective dynamics literature (Aldao, 2013; Koval et al., 2015), they are usually not taken into account as potentially time-structured. In the present chapter, I am concerned with explicit models for contextual processes and joint models for contextual and affective processes. Relying on work by Brandt and Sandler (2012), I propose a process model that accounts for the temporal structure in rates of (stressful) daily events. I also propose to combine this model with a Gaussian AR model to form a bivariate hybrid model for the joint dynamics of event rates and affective states. Simulations given smaller samples revealed good frequentist properties of the PAR model and largely satisfactory frequentist properties of the hPVAR model. I also showed, that a Gaussian AR model (with and without transformed process error) cannot recover the true parameter values well, when data are generated under the PAR model. Further, the PAR model and the hPVAR model both yielded better fit than their Gaussian counterparts to selected data from the COGITO study. With the bivariate model, however, I experienced problems with convergence, sampling efficiency (as was to some extent already the case in the simulation study), and theoretically implausible results. In the following, I would thus like to discuss potentials and limitations of the proposed modeling approach. The discussion features both technical and substantive aspects and includes some thoughts on future directions.

4.6.1 Potentials

A major potential of the models presented are the possibilities they offer to formalize and test ideas about the dynamics in daily events and about the dynamic interplay of daily events and affect. The bivariate model structure proposed may give rise to investigations of processes of such distinct nature as affective reactivity, affective anticipation, or processes of event evocation or avoidance. These processes plausibly contribute to daily affective functioning or emotion regulation in daily life, and therefore receive interest in the corresponding literature (e.g., Gross, 1998a). Also, the possibility to disentangle to what extent dynamics are located within a specific environment rather than within the person, puts popular narratives, which usually focus on intra-personal processes, to an empirical test.

In addition to these substantive implications, the present models also bear some potential along rather technical dimensions. First, I have demonstrated how employing the latent variable framework offers the possibility to link variables of different format. That is, by distinguishing latent and manifest variables, one can account for discrete-valued observed variables by continuous-valued latent variables – in this case event counts and event rates – via an appropriate measurement model. Change is then modelled at the level of the continuous-valued latent variables (see also Brandt et al., 2000). These continuous-valued process variables can then be linked to other continuous-valued variables, and their joint dynamics can be investigated, again, relatively independent of the format of any of the observed variables involved. One can hence integrate observed variables of different formats into a common model structure.

Second, the model implies a conditionally linear AR and CR structure (cf. Grunwald et al., 2000) and thus formulates additive relationships between latent process states over time. These additive effects transfer directly to the expectations at the manifest level. Such a model structure is in line with typically used linear VAR models. Reasons for committing to such a structure in the present context are that formal results concerning properties of the model-implied stochastic processes (e.g., stationarity), and the interpretation of model solutions may be borrowed from the linear VAR literature. Also, linear models might be a plausible starting point in the absence of strong prior knowledge on the form of daily dynamics and in the presence of noisy measurements.

Third, I rely on estimation by means of stochastic simulation using MCMC techniques, which is flexible and affords fitting complicated (hierarchical) models. I thereby also rely on the Bayesian framework. Still, I adopt the view that Bayesian *methods* can be used independently from Bayesian *philosophy of inference*, which is (supposedly) based on subjective beliefs (Gelman & Shalizi, 2013), and that Bayesian methods can even be used with frequentist goals (Wasserman, 2012). The intention to propose models that generalize beyond the specific application presented justifies "frequency calculations that calibrate Bayesian statements by tying them to frequencies of real-word events" (Rubin, 1984, p. 1153). In the simulation, I have therefore looked at frequentist accuracy and efficiency of the Bayesian estimators for one plausible set of parameter values. To pursue these lines of argumentation further, one may state that it is also not in contradiction with Bayesian modeling to engage in investigations of model fit and thus follow the established "hypothetico-deductive view of scientific method" (Gelman & Shalizi, 2013, p. 28). In the present application, I provide exemplary assessments of model (mis-)fit via posterior predictive checks at the level of the individual raw data. A major (yet to be realized) potential of the simulation-based estimation approach is that it is in principle relatively straightforward to formulate (and estimate) the proposed models in a fully hierarchical manner, hence allowing for IE differences in all IA model parameters. Such a setup would accommodate a greater variety of trajectory shapes, including the case of small-scale fluctuations at high levels of the process variables (i.e., the long-run mean and variance may be de-correlated across individuals; cf. Figure 4.3). Analogously to extending the model along the IE dimension, it may also be possible to do so along the IA dimension. For instance, one may want to test whether IA fluctuations are better accounted for by a heterogeneous process model that implies different long-run means of event rates for different time periods between which an individual switches. To get reliable solutions under such an approach, however, one would certainly require longer TS.

4.6.2 Limitations and future directions

Having started to elaborate on the to-be-realized potentials of the here-presented models, it is straightforward that I now move to the limitations.

First, the conditionally linear model structure I evoked (or rather kept) for reasons of consistence with the typical VAR literature, comes with certain problems. An issue of rather technical nature concerns potential inadmissible event rates under certain parameterizations of the presented model. When describing the model structures in Chapter 4, Section 4.2, I have noted that a sufficient condition for admissible event rate values is only met if, in addition, none of the AR or CR effects is negative. This somewhat unrealistic restriction also holds for Brandt and Sandler's observation-driven Poisson VAR model, although it is not discussed in the original paper. On the other hand, this is not a necessary condition, as negative AR and CR effects need not predict negative event rates, if the transformed process error has a high enough mean.

Another issue associated with the conditionally linear model structure is that I look at stressor pile-up in a linear way, that is, the hPVAR model postulates a linear effect of the number of events on negative affect. However, stressor pile-up is sometimes thought of as happening in a non-linear fashion in that the affective impact of an event may scale with the level of overall-stress (e.g., Schilling & Diehl, 2014). Such non-linear effects seem plausible, but may be hard to detect. While the linear model thus seems a good enough starting point, it could of course be extended to include such effects by making the effect of earlier events on affect dependent on the level of earlier stress. A bivariate threshold AR model (cf. De Haan-Rietdijk, Gottman, et al., 2016) may for example provide for a discretized formulation of such an idea.

A final issue in relation to model structure may arise due to the model implying a skewed long-run distribution not only for the event rate process, but also for the latent affect process. Although this feature of the model makes theoretical and empirical sense in the case of negative affect, it may require adjustment when the model is used in a different substantive context (e.g., with positive affect). A potential solution that comes to mind is to use alternative link functions. The $log(1 + exp(X))$ link (cf. Rasmussen & Nickisch, 2010), for example, behaves like a log link for (larger) negative values of X and like an identity link for (larger) positive values of X, and thus produces more symmetrical data. Additonally, incorporating the link function at another location in the model is possible, and also has implications for the problems mentioned above. Specifically, a generalized linear model-type specification that includes the log link in the measurement model does not require non-negative AR and CR effects to guarantee an admissible state space. At the same time, multiplicative effects would no longer be implied only within but also between time points, as a generalized linear model-type specification assumes multiplicative latent perturbations, so, perturbations whose size depends on the level of the latent outcome process.

A second set of limitations relates to the current implementation of the models in the Bayesian framework, which is definitely in need of refinement. So far, I have discussed MCMC techniques primarily as flexible and powerful estimation tools, but they also represent a complicated computational machinery that might unfold its full potential only when used in a relatively informed manner. For instance, I experienced some complications with general-purpose sampling algorithms implemented in JAGS. While the software offers some (automated) flexibility in terms of selecting different algorithms, the available samplers turned out to be largely inappropriate for estimating the hPVAR model in its original parameterization (i.e. they could not adapt). Using sampling algorithms more specifically tailored to the here-proposed models may not only facilitate or speed up adaptation, but may also lead to more efficient estimation and/or more accurate solutions. Inefficient estimation showed up as a particular problem of the bivariate model structures in the simulations (Chapter 4, Section 4.4.4), and especially in the application to COGITO data (Chapter 4, Section 4.5.4).

Further, Bayesian statistics rely essentially on the formulation of (hyper-)prior distributions for all model parameters. The usage of (informative) prior distributions can be conceived of as an opportunity for regularization and stabilization of results, for instance in small samples (e.g., Gelman & Shalizi, 2013; Zitzmann, Luedtke, & Robitzsch, 2015). In more complicated, multivariate models, where parameters are also likely to become differentially related conditional on the data, priors may impact modeling solutions in non-trivial ways and thus lead to unwanted biases. The unwanted and/or unforeseen biasing effects of priors pose a particular risk, if uninformative prior formulations are not generally available, as it is the case for variance parameters (e.g., Gelman, 2006; Schuurman, Grasman, et al., 2016). But also vague priors can have biasing effects if "the data do not contain enough information to override" them (McNeish, 2016a, p. 765). In the simulations in Chapter 4, Section 4.4, I reported differential patterns of parameter biases for the different model structures. As these biases seemed to be affected by sample size, it is likely that they reflect the impact of the prior distributions chosen, moderated by the respective model structures and associated parameter dependencies. For non-standard multivariate model structures, more systematic simulations can provide further insights. Specifically, one may vary the distributional moments of the presented priors and the prior distributions themselves to study the sensitivity of solutions to prior specifications. Alternatively, empirical Bayesian approaches may allow the specification of informative priors that introduce little bias (e.g., McNeish, 2016b).

Finally, model fit could be investigated more rigorously. While I presented only a heuristic version of posterior predictive checks, rather ranging in the league of face validations, it is possible to derive sensible aggregations of the data in terms of test statistics and associated posterior predictive p-values.

To leave the territory of mainly technical criticism, I would now like to turn to a general substantive limitation that concerns the treatment of negative events as cumulative and therefore exchangeable. As elaborated on earlier, the investigation of the frequency of specific events might qualify as an investigation of more "objective" aspects of stress (Almeida, 2005, p. 65). This seems not only a theoretically reasonable angle of looking at stress (Kanner et al., 1981), but also appears sensible from a measurement perspective. That is, by asking for events in terms of their abstract qualities instead of in terms of their detailed, individual perception, I probably minimize conceptual overlap with affect measures (cf. Montpetit et al., 2010) and thus the risk of measurement bias (i.e., being more likely to report a negative event or an event as negative just because being in a more negative affective state). The potential downside of this is of course that individuals can perceive a given event in different ways, and exactly this individualized flavor of an event might be as important as or more important for affective functioning than information on just event occurrence (e.g., Almeida et al., 2002). This relates to the question of "whether the impact of a hassle depends merely on its cumulative impact or on its content and meaning in the person's life" (Kanner et al., 1981, p. 5). In the present setup, I even treat different events from different life domains as exchangeable. Whereas the estimation of individual-specific effects of single events seems to bear problems of model identification, one could think of evoking a common latent variable that accounts for common changes in the rates of events from different domains. It could then be investigated to what extent the effect of daily stressors of different quality on affective states can be accommodated by a common latent stressor variable and to what extent stressful events from specific domains have specific effects on affect.

As a very final point, the statements that the present model could serve to, first, disentangle affective and contextual dynamics, and, second, identify the nature of processes involved in reciprocal person-situation interactions, might require some qualification. First, events are often, as in the COGITO data, measured by self-report, and are therefore not truly objective. In an extreme case, changes and dynamics in self-reported negative events might be driven *completely* by an individual's perceptions of the events and might thus be an intra-personal process as well. Still, it may remain a process that is sufficiently specific and thus potentially distinct and distinguishable from a process of negative affect. Here, I assume that measurement bias as described above (i.e., measures of stressor occurrence also assessing negative affect to some extent) is more likely to occur within than between time points, and will thus not primarily affect the estimation of time-lagged dynamic effects. In case there are both subjective and objective or at least subjective and person-independent information on context available (e.g., other-reports, cf. Almeida et al., 2002), it might be interesting to link the two streams of contextual information, and to also quantify the extent to which independently assessed contextual variations have direct effects on affective functioning versus indirect effects, mediated via an individual's perceptions and interpretations.

Second, although the bivariate hybrid model allows distinguishing cross-lagged associations in both directions between the two processes, specific attributions of effects to processes of situation anticipation, evocation, or selection, for example, remain (competing) interpretations without a guarantee to be true. As an example, consider the case where other actors have much more control over the daily events in a person's environment than the person him- or herself. High effects of earlier affect on later events might then indicate reactivity of the environment, rather than active acts of situation anticipation, evocation, or even selection by the person him- or herself. Hence, the nature of statistical associations, also of IA associations, remains to some extent ambiguous in uncontrolled and sparse observational micro-longitudinal data.

5 Discussion

I described the rise of what I have termed the *micro-longitudinal paradigm* in affect research (see also Hamaker & Wichers, 2017), relying on intense longitudinal measurements and modeling techniques suitable to extract potential temporal dynamics from the obtained data. I have argued that the paradigms' recent popularity is driven by the motivation to learn about the processes of daily affective functioning, such as emotion regulation processes. These processes are of interest on their own, but also when it comes to understanding emotional development.

The present thesis is concerned with *contextualizing* affective dynamics, and specifically with modeling approaches that formalize affective functioning as *embedded in* and *interacting with* daily contextual variations. With its methodological focus, this piece thus ties into the rather data-driven body of work the field has produced so far.

After a brief summary of the presented methodological contributions, I devote the remainder of the thesis to a discussion of more general conceptual issues. The discussion is structured such that I revisit the themes introduced in Chapter 2, moving from the more thesis-specific to the more general problems and ideas.

5.1 Thesis summary

5.1.1 Fixed moderated time series analysis

As a first contribution, we (Adolf et al., 2017) presented *fixed moderated time series analysis* (fmTSA) to estimate context-related changes in the parameters of a dynamic time series (TS) model. The model estimates the amount of parameter change that follows a known shape, and this shape can be determined by observed contextual changes. Additionally, it is possible to formulate and test primarily time-related parameter changes such as trends. Statistically, this line of work revolves around the problem of intra-individual (IA) heterogeneity, that is, the situation in which more than one set of dynamic parameters is required to characterize a processes' behavior over time. With fmTSA, we treat IA heterogeneity as observed, which is a major difference to existing parametric modeling solutions, such as regime switching modeling. We have shown by simulation that the approach (i.e., the ML interval estimates) can perform sufficiently accurate in smaller samples. Also, the model is relatively easy to implement in standard software, in this case via the Kalman filter implementation of the free and open source software OpenMx (Boker et al., 2015; Hunter, 2014b; Neale et al., 2015). The model is applicable to data from single individuals, allowing for the unconstrained exploration of generalities and specificities among individual affective dynamics in context. Drawbacks are that *some* knowledge about potential parameter changes is required and that the moderator itself is not modelled over time. This implies that uncertainty in the shape of parameter change as well as measurement error in and missing observations on the moderator cannot directly be accounted for. Also, to what extent contextual changes are time-structured themselves, and might even be affected by earlier affective states, cannot be estimated. A detailed discussion of the potentials and limitations of the approach was provided in Chapter 3, Section 3.5 (cf. Adolf et al., 2017). Additionally, I outlined a potential application of the model to experimental data generated in a virtual reality.

5.1.2 Poisson and Gaussian vector autoregressive modeling of context and affect

As a second contribution, I proposed a dynamic model for changes in daily events as a typical instantiation of daily context. By adopting and modifying a *Poisson autoregressive* (PAR) model that estimates dynamic effects at the level of latent event rates (cf. Brandt & Sandler, 2012), I implement a process perspective on daily events. This process perspective offers a take on contextual dynamics on their own, but also in interaction with affective dynamics. To achieve the latter, and thereby draw a more differentiated picture of daily affective functioning, I set up a *hybrid Poisson-Gaussian vector autoregressive* (hPVAR) model for daily events and affective states. In typically uncontrolled intensive longitudinal data, teasing contextual and intra-personal dynamics apart by statistical modeling may also allow for more specific attributions of dynamic effects. I again support these modeling suggestions by simulations, and an application to selected data from the COGITO study. Potentials and limitations were discussed in length in Chapter 4.6. On the positive side, these involve, besides the possibility to formalize reciprocal person-situation interactions, the linkage to the well-established vector autoregressive (VAR) literature, and the estimation via flexible and powerful stochastic simulation techniques. On the negative side, I discuss model structure-related limitations and complications of the implementation in the Bayesian framework. Future substantive research could investigate whether daily affective experiences interact differentially with stressors from different domains, or with subjectively versus (more) objectively measured events.

5.2 General discussion

5.2.1 Perspectives on contextualizing affective dynamics

To motivate and complement the methodological work in the main body of this thesis, I have initially presented different stances one might take regarding the role of context for inquiries into affective functioning. These stances offer – at different levels of abstraction – arguments for why it might be worthwhile or even necessary to look at *contextualized* affective dynamics.

Originating in the person-situation view (and later debate) in personality psychology (Epstein & O'Brien, 1985; Kenrick & Funder, 1988; Lewin, 1936), procedural personality theories (e.g., Cervone, 2004) view contextual or environmental factors as an integral part of the individual as a psychological system. A similar position is also taken by dynamical systems theory (e.g., Bergman & Wångby, 2014; Thelen, 2005; van Geert & Steenbeek, 2005), which is rooted in developmental psychological traditions.

In characterizing these general substantive stances, I distinguished two notions that could drive the specification of person-situation interactions. Whereas the *hierarchical notion* conceives of psychological functioning as embedded in and thus unfolding conditional on situations, the *reciprocal notion* focusses more on dynamic exchanges between persons and situations. Note that these notions are of rather heuristic, non-formal nature, and are therefore not meant to be (mutually) exclusive.

Assuming that affective functioning is embedded in context and/or, interacts reciprocally with context, the statistical stance holds that *failing* to take context into account can lead to ambiguous, biased or even spurious effects and false conclusions regarding affective dynamics. *Taking* context into account, on the contrary, can lead to more holistic and differentiated descriptions of daily affective life. Conceiving of affective functioning as nested within situations enables studying *systemic* reactions to contextual changes. For instance, one could represent individuals as being *flexible* in their affective dynamics, and hence potentially in their emotion regulation activities, contingent upon situational demands (e.g., Aldao, 2013; De Haan-Rietdijk, Gottman, et al., 2016; Sliwinski et al., 2009). Taking a slightly different angle and looking at reciprocal interactions between affective and contextual processes as *complementary* processes allows getting at the unique and joint dynamics of contextual and affective changes. This may lead to more nuanced descriptions of the *bi-directional* transactions between context and affect in daily life, potentially enabling the distinction of processes such as affective reactivity from the anticipation, evocation, and selection of situations.

Further refining the understanding of daily affective functioning also concerns the question of how contextualized affective dynamics feature in the emergence and stabilization of inter-individual (IE) differences and thus give rise to longer-term emotional development (Bos & De Jonge, 2014; Wichers et al., 2015). Here, especially the above described systemic take can yield a central contribution to understanding how adaption – in terms of the long-term optimization of global emotional outcomes – to *variable* environments manifests in *variable* patterns of experiences and behaviors (cf. Gluck et al., 2012).

Incorporating (existing) contextual information into modeling is of course an important requirement for such inquiries. However, in theoretical and measurement-theoretical regards, one might need to take one step back in order to take two steps forward: Whereas researchers have recently embarked on reviewing and integrating findings on affective dynamics, often in relation to contextual variations (e.g., Aldao et al., 2015; Bonanno & Burton, 2013; Hollenstein et al., 2013; Kashdan & Rottenberg, 2010), the resulting taxonomies need yet to be condensed into testable theories and quantifiable models. For instance, based on a review of experimental and observation affect research, Bonanno and Burton develop an idea of "regulatory flexibility" (Bonanno & Burton, 2013, p. 591 ff.). They argue that regulatory activities cannot be adaptive or maladaptive per se, but that their "efficacy" also depends on situational fit, and conceptualize regulatory flexibility as consisting of three sequential components that may achieve person-situation fit, namely, "context sensitivity", regulatory "repertoire", and responsiveness to feedback during the monitoring of regulatory behavior (Bonanno & Burton, 2013, p. 595 ff.). The proposed model is reminiscent of other process models of emotion regulation (Gross, 2015) and seems to possess mechanistic, generative potential. However, it capitalizes on qualitative distinctions that do not only require appropriate statistical models, but – in the first place – appropriate operationalizations and data.

So, the conceptualization and measurement of various affective experiences and behaviors (Cole et al., 2004; Gross, 2015), which I come back to in the next chapter, but also the conceptualization and measurement of context seem to require some (re-)focus in future work. How to research situations arises as a general topic in the personality psychology literature (Horstmann, Rauthmann, & Sherman, accepted; Rauthmann, 2015; Rauthmann, Sherman, & Funder, 2015). Here, critical and open questions concern the decomposition of situational information, the tension between "objectivist" and "subjectivist" perspectives, and principles that could structure and unify future scientific effort in this area (Rauthmann et al., 2015, p. 364 ff.).

5.2.2 The psychological substance of the micro-longitudinal paradigm

The present thesis draws heavily on the recent enthusiasm in relation to intense longitudinal data, dynamic models and derived conceptualizations of affective phenomena (Hamaker et al., 2015; Hamaker & Wichers, 2017; Kuppens & Verduyn, 2015; Röcke & Brose, 2013). While I consider this a fruitful development, there are some complications worth keeping in mind.

How well "intrinsic psychological properties" (Boker, 2002, p. 405) can be extracted and hence *genuine* regulatory principles can be identified from the parameters of dynamic models fitted to repeated measures of affective states depends on the quality of both the data and the models at hand. A too liberal usage of narratives such as the one of "emotional inertia" (e.g., Kuppens, Allen, et al., 2010) suggests that describing IA variation almost *guarantees* insights into specific psychological processes. In the following, I list supposed threats to such a "parameter realism", where one, in the extreme case, might be tempted to interpret parameters as directly reflective of intra-personal properties or kinds.

A methodological concern that has already been mentioned in Chapter 4 relates to how the ecologically valid, but uncontrolled nature of daily diary or experience sampling data complicates the interpretation of dynamic effects. It is to be expected that affective states do not occur independent of other intra-personal states, but also not independent of the environment an individual functions in. And changes in all of these domains might well be time-structured. Specific (causal) attributions of dynamic effects (e.g., as specific intra-personal effects) are then only warranted to the extent the different processes – and specific contextual processes are again only one example – can be disentangled. To improve this, assessment and statistical modeling need to become more comprehensive. Alternatively, micro-longitudinal studies may be combined with interventions, bringing parts of the system under control (Hamaker & Wichers, 2017; Voelkle, 2017). Also affective functioning may be studied in emotionally loaded virtual realities (e.g., McCall et al., 2016), which may serve as internally and externally valid minimal models of daily contexts.

Concerns may also be voiced with respect to conceptual and operational decisions. Few scholars would probably question that subjective affective states, which are amenable to introspection and self-report, are not only "real" (Barrett, 2012, p. 413), but also meaningful psychological entities (van de Leemput et al., 2014) in that they are systematically related to processes of emotion regulation. However, do reportable affective states allow on-line tracking of regulatory processes or do they provide a rather indirect account of emotion regulation activities in the sense that they only reflect accumulated regulatory outcomes and are therefore not suited to capture regulatory dynamics (cf. Cole et al., 2004)? Or, alternatively, do affective self-reports maybe “blur the distinction between [initial] appraisal and regulatory strategy” (Bonanno & Burton, 2013, p. 598)? Distinguishing processes of emotion regulation from their (local) antecedents and outcomes seems a crucial question when searching for adaptive patterns of affective functioning, because adaptive patterns may look very different for the different stages. As Gluck and colleagues (2012, p. 201) put it, “flexibility of behavior may contribute to the robustness of a fitness outcome”, where “behavior” may refer to regulatory activities and the robust (i.e., invariant) “outcome” may refer to a global emotional state such as well-being – but also a local emotional state such as momentary satisfaction. Also here, the aim should be to strengthen the feedback loop from empirical to conceptual to empirical work, thereby hopefully “asking successively better questions, and using successively more refined empirical and theoretical approaches” (Gross, 2015, p. 20).

Another issue with respect to measurement arises if one puts the emphasis on the individual as the unit of analysis. Then, problems of systematic measurement error (with respect to individuals) and validity of measures seem to delineate an interesting area of conflict (cf. Adolf et al., 2014). Here, discussions about the trade-off between individual meaningfulness and generalizability of measurement procedures (e.g., Borsboom & Dolan, 2007; Nesselroade, Gerstorf, Hardy, & Ram, 2007; Nesselroade, Ram, Gerstorf, & Hardy, 2009) await further resolution.

5.2.3 The individual as the unit of analysis

The importance of choosing the level of analysis that matches the phenomenon one is interested in, in this case the IA level, is evident and seems generally acknowledged. It is also clear that this does not prevent aggregations at the next level of hierarchy, in this case the IE level (rather the contrary as some have argued; e.g., Lindenberger & Von Oertzen, 2006; P. C. M. Molenaar, 2004; Nesselroade, 2010).

In general, a *statistical* integration of the IA and the IE level of analysis can be carried out in a bottom-up direction from the individual to the population or in a top-down direction, from the population to the individual using hierarchical modeling approaches.

When a bottom-up strategy is pursued and individual cases are analyzed independently, as in Chapter 3 (cf. Adolf et al., 2017), the question arises as to how one can optimally capitalize on a person-centered approach. Whereas I used one and the same model structure for each person and then described the distribution of effects across individuals, person-specific modeling offers much more *explorative* capacity. To determine individual-specific model structures, more data-driven approaches might be used, for instance. This may either include techniques from the field of statistical learning that automate the search for systemic patterns in the data, while controlling for decision errors and overfitting, or a smaller-scale theory-guided search among plausible or interesting parametric models via model diagnosis and model selection. Given individual-specific model structures, an important question then becomes how to compare the solutions across individuals at a more abstract level. What are useful concepts that "supervene" (cf. Borsboom et al., 2009, p. 23 ff.) in a statistical sense? One example popular in psychological applications is graph theory and according measures of model structure (e.g., centrality; Epskamp, 2016). In the field of clinical and personality psychology, a wealth of studies has employed these theories and measures lately (Borsboom, Cramer, Schmittmann, Epskamp, & Waldorp, 2011; Bringmann et al., 2013; Cramer et al., 2016; Pe et al., 2015; Schmittmann et al., 2013). Alternatively to addressing model structures directly, it is possible to draw comparisons at the level of the model-implied probability distributions. One way to do this is by using information theoretic measures (e.g., Epskamp, 2016; Schmiedek et al., 2017). As a descriptive alternative, Ram and colleagues (Ram et al., 2013) adapted taxonomies used in geography to characterize and compare individual distributions in terms of their landscape-like properties.

When a top-down strategy is pursued, the statistical integration of the IA and the IE level of analysis is already implied. Here, IA and IE differences are modelled simultaneously and the IE model constrains or, to put it more neutral, informs the IA results. However, I have already alluded to the idea of a continuum between bottom-up and top-down approaches. This becomes obvious when parametric models are considered, which are still a standard in psychology. In such cases, there will always – also under a bottom-up approach – be a general model structure that is – at some level of complexity – common to individuals. If this is then combined with flexible Bayesian modeling techniques that are not limited in terms of distributional assumptions regarding (hyper-)priors, it seems as if single-subject modeling and hierarchical modeling approaches become very similar. Arguments that capitalize on the dichotomy between bottom-up versus top-down strategies may have to be revised in the increasing usage of flexible Bayesian modeling approaches.

From a very practical point of view, the suggestion that "unbiased descriptions and explanations of the differences and commonalities among" IA patterns of variation might be more likely the more bottom-up the analysis approach (Lindenberger & Von Oertzen, 2006, p. 300) is not necessarily correct, though. With finite data, a condition I put a major emphasis on in the simulations, one seems to trade different biases as one moves along the continuum between bottom-up and top-down. Given individual TS models and ML estimation, problems of finite sample biases arise. That is, in Chapter 3, Section 3.3.2, I report finite sample biases, for instance in autoregressive (AR) parameters, that are substantial for short TS (cf. Adolf et al., 2017). Problems of finite sample bias in TSA seem to be well known in the econometric and statistical literature (e.g., Cheang & Reinsel, 2000; Maeshiro, 2000), but less acknowledged in the psychometric literature. In hierarchical models, distributional assumptions at the population level can bias (or regularize) results at the individual level. This also holds for prior and hyperprior distributions as evoked in a Bayesian framework. Given that (behavioral) micro-longitudinal studies cannot not produce unlimited data, such trade-offs, which also concern estimation precision and power (e.g., von Oertzen & Boker, 2010), require some careful consideration along substantive and statistical lines.

An integration of the IA and the IE level of analysis does not only require appropriate statistical techniques, but also *theoretical* effort, in that concepts evoked at both levels need to be linked. Attempts to statistically relate global emotional outcomes to specific IA dynamics of affective functioning across individuals are prevalent in the current literature on affective functioning. And so are the corresponding theoretical ideas to explain successful development and adaption via daily affective dynamics[11]. So far, I have argued in the spirit of the empirical problem-reasoning discussed in Chapter 2, Section 2.1.2. However, following arguments reviewed in Chapter 2, Section 2.1.3, one might wonder how sensible the search for *generalizable* patterns of adaptive affective functioning is, how likely it is to boil high well-being or low depression down to specific patterns of functioning across individuals. An abstract concept such as adaption or successful development might be intractable at the IA level (Borsboom et al., 2009). For instance, environments of a certain complexity do not come with specific, well-defined adaptive problems, so, selection pressures might be rather diffuse. Consequently, there may exist multiple alternative optimal solutions to multiple adaptive

[11] It has been argued that, to empirically support such ideas, one would actually have to link earlier *changes* in short-term dynamics to later *changes* in long-term development(cf. Adolf, Voelkle, Brose, & Schmiedek, 2017; Bos & De Jonge, 2014; Wichers, Wigman, & Myin-Germeys, 2015)

problems. Moreover, existing solutions need not even be optimal, but sub-optimal ones can still be sufficient, extending the space of possible solutions even further. So, there is much "wiggle room" in how adaptiveness may manifest given the complexity of individuals and the contexts they live in.

5.3 Outlook

Surveying the recent observational affect literature, there seems to exist considerable enthusiasm about the micro-longitudinal paradigm holding potential for understanding affective functioning in everyday life. Following recent calls in the theoretical literature to acknowledge the importance of context, I have focused on methodological approaches to incorporate changing contextual states into dynamic models of affective functioning.

In connection to a relatively data-driven body of work, this seems a valid first step to approach contextualized affective dynamics. The concluding discussion has shown, however, that formal dynamic models in application to observational micro-longitudinal data do not necessarily guarantee insights into psychological processes. Also, the current micro-longitudinal affect research may benefit from being complemented by more intense theoretical work on emotion regulation in context.

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Appendices

A Long-run latent process moments for a fixed moderated time series model

In the following, I derive the marginal long-run probability distribution implied for the latent process by the model presented in Equations (3.3) and (3.4) over a dichotomous, dummy coded moderator. Remember that the latent process is defined as

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha}^{(0)} + X_{t-1}\boldsymbol{\alpha}^{(X)} + \left(\mathbf{B}^{(0)} + X_{t-1}\mathbf{B}^{(X)}\right)\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \tag{A.1}$$

with

$$\boldsymbol{\zeta}_t \sim N\left(\mathbf{0}, \boldsymbol{\Psi}^{(0)} + X_{t-1}\boldsymbol{\Psi}^{(X)}\right)$$

and

$$X_t = x_t \in (0,1).$$

Given stationarity conditional on the moderator, so, given

$$\mathbb{E}(\boldsymbol{\eta}_t | X_{t-1} = 0) = \mathbb{E}(\boldsymbol{\eta}_{t-1} | X_{t-2} = 0),$$
$$\mathbb{E}(\boldsymbol{\eta}_t | X_{t-1} = 1) = \mathbb{E}(\boldsymbol{\eta}_{t-1} | X_{t-2} = 1),$$

and

$$\mathbb{E}\left((\boldsymbol{\eta}_t | X_{t-1} = 0)(\boldsymbol{\eta}_t | X_{t-1} = 0)^{\mathrm{T}}\right) = \mathbb{E}\left((\boldsymbol{\eta}_{t-1} | X_{t-2} = 0)(\boldsymbol{\eta}_{t-1} | X_{t-2} = 0)^{\mathrm{T}}\right),$$
$$\mathbb{E}\left((\boldsymbol{\eta}_t | X_{t-1} = 1)(\boldsymbol{\eta}_t | X_{t-1} = 1)^{\mathrm{T}}\right) = \mathbb{E}\left((\boldsymbol{\eta}_{t-1} | X_{t-2} = 1)(\boldsymbol{\eta}_{t-1} | X_{t-2} = 1)^{\mathrm{T}}\right),$$

I can derive the moments of the model-implied long-run distribution conditional on the moderator.

The conditional means are

$$
\begin{aligned}
\boldsymbol{\nu}^{(0)} &= \mathbb{E}(\boldsymbol{\eta}_t | X_{t-1} = 0) \\
&= \mathbb{E}\left(\boldsymbol{\alpha}^{(0)} + \mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1} | X_{t-2} = 0) + \boldsymbol{\zeta}_t\right) \\
&= \boldsymbol{\alpha}^{(0)} + \mathbf{B}^{(0)}\boldsymbol{\nu}^{(0)} \\
\Leftrightarrow \boldsymbol{\alpha}^{(0)} &= \boldsymbol{\nu}^{(0)} - \mathbf{B}^{(0)}\boldsymbol{\nu}^{(0)} = \left(\mathbf{I} - \mathbf{B}^{(0)}\right) \boldsymbol{\nu}^{(0)} \\
\Leftrightarrow \boldsymbol{\nu}^{(0)} &= \left(\mathbf{I} - \mathbf{B}^{(0)}\right)^{-1} \boldsymbol{\alpha}^{(0)}
\end{aligned}
\tag{A.2}
$$

and

$$
\begin{aligned}
\boldsymbol{\nu}^{(X)} &= \mathbb{E}(\boldsymbol{\eta}_t | X_{t-1} = 1) \\
&= \mathbb{E}\left(\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} + (\mathbf{B}^{(0)} + \mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1} | X_{t-2} = 0) + \boldsymbol{\zeta}_t\right) \\
&= \boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} + \left(\mathbf{B}^{(0)} + \mathbf{B}^{(X)}\right)\boldsymbol{\nu}^{(X)} \\
\Leftrightarrow \boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} &= \boldsymbol{\nu}^{(X)} - \left(\mathbf{B}^{(0)} + \mathbf{B}^{(X)}\right)\boldsymbol{\nu}^{(X)} = \left(\mathbf{I} - \left(\mathbf{B}^{(0)} + \mathbf{B}^{(X)}\right)\right) \boldsymbol{\nu}^{(X)} \\
\Leftrightarrow \boldsymbol{\nu}^{(X)} &= \left(\mathbf{I} - \left(\mathbf{B}^{(0)} + \mathbf{B}^{(X)}\right)\right)^{-1}\left(\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)}\right).
\end{aligned}
\tag{A.3}
$$

The conditional variance-covariance matrices are

$$
\begin{aligned}
\mathbf{P}^{(0)} &= \mathbb{E}\left((\eta_t|X_{t-1}=0)(\eta_t|X_{t-1}=0)^{\mathrm{T}}\right) \\
&= \mathbb{E}\left(\left(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)+\boldsymbol{\zeta}_t\right)\left(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)+\boldsymbol{\zeta}_t\right)^{\mathrm{T}}\right) \\
&= \mathbb{E}\left(\left(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)+\boldsymbol{\zeta}_t\right)\left((\boldsymbol{\eta}_{t-1}|X_{t-2}=0)^{\mathrm{T}}\mathbf{B}^{(0)^{\mathrm{T}}}+\boldsymbol{\zeta}_t^{\mathrm{T}}\right)\right) \\
&= \mathbb{E}\left(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)^{\mathrm{T}}\mathbf{B}^{(0)\mathrm{T}}+\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t^{\mathrm{T}}\right) \\
&= \mathbf{B}^{(0)}\mathbf{P}^{(0)}\mathbf{B}^{(0)^{\mathrm{T}}}+\boldsymbol{\Psi}^{(0)} \\
\Leftrightarrow vec\left(\mathbf{P}^{(0)}\right) &= \left(\mathbf{B}^{(0)}\otimes\mathbf{B}^{(0)}\right)vec\left(\mathbf{P}^{(0)}\right)+vec\left(\boldsymbol{\Psi}^{(0)}\right) \\
\Leftrightarrow vec\left(\boldsymbol{\Psi}^{(0)}\right) &= vec\left(\mathbf{P}^{(0)}\right)-\left(\mathbf{B}^{(0)}\otimes\mathbf{B}^{(0)}\right)vec\left(\mathbf{P}^{(0)}\right)=\left(\mathbf{I}-\left(\mathbf{B}^{(0)}\otimes\mathbf{B}^{(0)}\right)\right)vec\left(\mathbf{P}^{(0)}\right) \\
\Leftrightarrow vec\left(\mathbf{P}^{(0)}\right) &= \left(\mathbf{I}-\left(\mathbf{B}^{(0)}\otimes\mathbf{B}^{(0)}\right)\right)^{-1}vec\left(\boldsymbol{\Psi}^{(0)}\right)
\end{aligned}
\tag{A.4}
$$

and

$$
\begin{aligned}
\mathbf{P}^{(X)} &= \mathbb{E}\Big((\eta_t|X_{t-1}=1)(\eta_t|X_{t-1}=1)^{\mathrm{T}}\Big)\\
&= \mathbb{E}\left(\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)\boldsymbol{\zeta}_t\right)\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)+\boldsymbol{\zeta}_t\right)^{\mathrm{T}}\right)\\
&= \mathbb{E}\left(\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)+\boldsymbol{\zeta}_t\right)\left((\boldsymbol{\eta}_{t-1}|X_{t-2}=1)^{\mathrm{T}}\left(\mathbf{B}^{0}+\mathbf{B}^{(X)}\right)^{\mathrm{T}}+\boldsymbol{\zeta}_t^{\mathrm{T}}\right)\right)\\
&= \mathbb{E}\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)^{\mathrm{T}}\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)^{\mathrm{T}}+\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t^{\mathrm{T}}\right)\\
&= \left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\mathbf{P}^{(0)}\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)^{\mathrm{T}}+\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)} \qquad \text{(A.5)}\\
\Leftrightarrow vec\left(\mathbf{P}^{(X)}\right) &= \left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\otimes\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\right)vec\left(\mathbf{P}^{(X)}\right)+vec\left(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)}\right)\\
\Leftrightarrow vec\left(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)}\right) &= vec\left(\mathbf{P}^{(X)}\right)-\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\otimes\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\right)vec\left(\mathbf{P}^{(X)}\right)\\
&= \left(\mathbf{I}-\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\otimes\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\right)\right)vec\left(\mathbf{P}^{(X)}\right)\\
\Leftrightarrow vec\left(\mathbf{P}^{(X)}\right) &= \left(\mathbf{I}-\left(\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\otimes\left(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}\right)\right)\right)^{-1}\left(vec\left(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)}\right)\right).
\end{aligned}
$$

In general, the moments of a finite mixture of multivariate normal distributions are

$$\mathbf{v}_{mix} = \sum_{r=1}^{\mathrm{R}} w_r \boldsymbol{v}_{\mathrm{r}} \tag{A.6}$$

and

$$\mathbf{P}_{mix} = \sum_{r=1}^{\mathrm{R}} w_r \mathbf{P}_{\mathrm{r}} + \sum_{r=1}^{\mathrm{R}} w_r (\boldsymbol{v}_{\mathrm{r}} - \boldsymbol{v}_{mix})(\boldsymbol{v}_{\mathrm{r}} - \boldsymbol{v}_{mix})^{\mathrm{T}} \tag{A.7}$$

with w_r being relative weights summing to one (Hardy, 2012). In the case of a dummy coded dichotomous moderator, this is

$$\mathbf{v}_{mix} = (1 - \bar{X})\mathbf{v}^{(0)} + \bar{X}\mathbf{v}^{(X)} \tag{A.8}$$

and

$$\mathbf{P}_{mix} = (1 - \bar{X})\mathbf{P}^{(0)} + \bar{X}\mathbf{P}^{(X)} + (1 - \bar{X})\left(\mathbf{v}^{(0)} - \mathbf{v}_{mix}\right)\left(\mathbf{v}^{(0)} - \mathbf{v}_{mix}\right)^{\mathrm{T}} + \bar{X}\left(\mathbf{v}^{(X)} - \mathbf{v}_{mix}\right)\left(\mathbf{v}^{(X)} - \mathbf{v}_{mix}\right)^{\mathrm{T}}. \tag{A.9}$$

B OpenMx script for a fixed moderated time series model

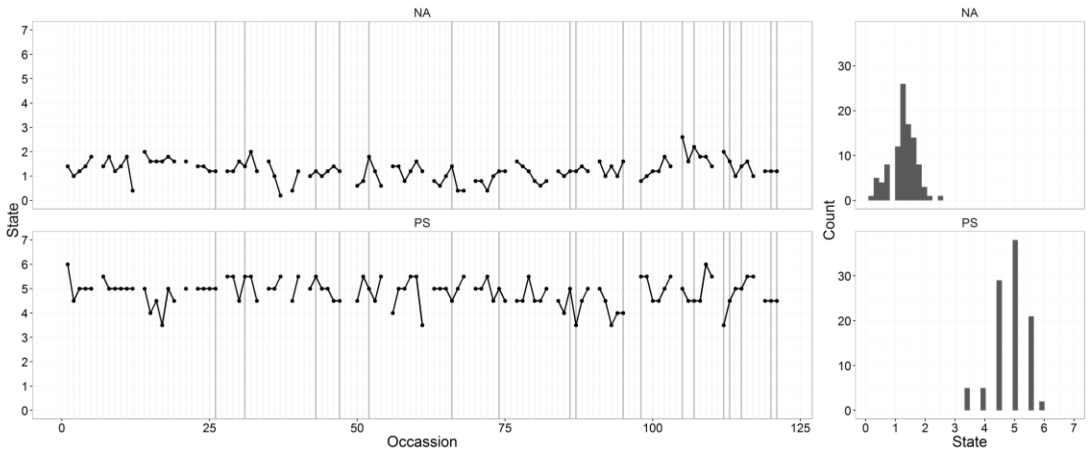
```
## load package OpenMx
library(OpenMx)

## set NPSOL as default optimizer
if (options()$mxOption$'Default optimizer'!='NPSOL'){ mxOption(NULL, "Default optimizer", 'NPSOL')}

## get data
data <- get(load('exampledata.rda'))
name.y <- c('y1','y2') # names in data
name.x <- 'xlagged' #
name.c <- 'constant' #

## define parameter (starting) values; B is Beta, a is alpha,…
B.init <- matrix(c(.3, 0, 0, .3), 2, 2)
a.init <- matrix(c(1, 1), 2, 1)
P.init <- matrix(c(.3, 0, 0, .3), 2, 2)
BX.init <- matrix(c(0, 0, 0, 0), 2, 2)
aX.init <- matrix(c(0, 0), 2, 1)
PX.init <- matrix(c(0, 0, 0, 0), 2, 2)
L.init <- diag(2)
ta.init <- matrix(0, 2, 1)
Th.init <- diag(2)*0

## define which parameters are freely estimated and which are fixed
B.est <- matrix(TRUE, 2, 2) # freely estimated
a.est <- matrix(TRUE, 2, 1)
P.est <- matrix(TRUE, 2, 2)
BX.est <- matrix(TRUE, 2, 2)
aX.est <- matrix(TRUE, 2, 1)
PX.est <- matrix(TRUE, 2, 2)
L.est <- matrix(FALSE, 2, 2) # fixed
ta.est <- matrix(FALSE, 2, 1)
Th.est <- matrix(FALSE, 2, 2)

## define parameter labels
B.label <- matrix(c(paste0(paste0('b', 1:2), rep(1:2, each=2))), 2, 2)
a.label <- matrix(paste0('a', 1:2), 2, 1)
P.label <- matrix(c(paste0(paste0('p', 1:2), rep(1:2, each=2))), 2, 2)
P.label[upper.tri(P.label)] <- P.label[lower.tri(P.label)] # make label matrix symmetric as this is a covariance matrix
BX.label <- matrix(c(paste0(paste0('bx', 1:2), rep(1:2, each=2))), 2, 2)
aX.label <- matrix(paste0('ax', 1:2), 2, 1)
PX.label <- matrix(c(paste0(paste0('px', 1:2), rep(1:2, each=2))), 2, 2)
PX.label[upper.tri(PX.label)] <- PX.label[lower.tri(PX.label)] # make label matrix symmetric as this is a covariance matrix
```

```
L.label <- matrix(c(paste0(paste0('l', 1:2), rep(1:2, each=2))), 2, 2)
ta.label <- matrix(paste0('ta', 1:2), 2, 1)
Th.label <- matrix(c(paste0(paste0('th', 1:2), rep(1:2, each=2))), 2, 2)
Th.label[upper.tri(Th.label)] <- Th.label[lower.tri(Th.label)]  <- NA # make label matrix diagonal as this is a covariance matrix

## define initial moments to start the Kalman filter
x0.init  <-  matrix(apply(as.matrix(data[, name.y]), 2, mean, na.rm=T), 2, 1)
P0.init  <-  matrix(cov(as.matrix(data[, name.y]), use='pairwise.complete.obs'), 2, 2)

## define MxModel and store into object 'model'
model <- mxModel(
  name='model',
  ## definition variables
  mxMatrix("Full", nrow=1, ncol=1, free=FALSE, labels=matrix(paste0("data.", name.c), 1, 1), name="u"),
  mxMatrix("Full", nrow=1, ncol=1, free=FALSE, labels=matrix(paste0('data.', name.x), 1, 1), name='X'),
  ## measurement model
  mxMatrix("Full", nrow=2, ncol=2, values=L.init, free=L.est, labels=L.label, name="L", dimnames=list(name.y, paste0("latent", name.y))),
  mxMatrix("Full", nrow=2, ncol=1, values=ta.init, free=ta.est, labels=ta.label, name="ta"),
  mxMatrix("Diag", nrow=2, ncol=2, values=diag(Th.init), free=diag(Th.est), labels=diag(Th.label), name="Th", lbound=diag(2)*0),
  ## latent process model
  mxMatrix("Full", nrow=2, ncol=2, values=B.init, free=B.est, labels=B.label, name="B"),
  mxMatrix("Full", nrow=2, ncol=1, values=a.init, free=a.est, labels=a.label, name="a"),
  mxMatrix("Full", nrow=2, ncol=2, values=P.init, free=P.est, labels=P.label, name="P", lbound=matrix(c(0, -10, -10, 0), 2, 2)),
  mxMatrix("Full", nrow=2, ncol=2, values=BX.init, free=BX.est, labels=BX.label, name="BX"),
  mxMatrix("Full", nrow=2, ncol=1, values=aX.init, free=aX.est, labels=aX.label, name="aX"),
  mxMatrix("Full", nrow=2, ncol=2, values=PX.init, free=PX.est, labels=PX.label, name="PX"),
  ## define Algebras
  mxAlgebra(B+BX%x%X, name="Bstar"),
  mxAlgebra(a+aX%x%X, name="astar"),
  mxAlgebra(P+PX%x%X, name="Pstar"),
  ## Kalman Filter initialization
  mxMatrix("Full", nrow=2, ncol=1, values=x0.init, free=FALSE, name="x0"),
  mxMatrix("Full", nrow=2, ncol=2, values=P0.init, free=FALSE, name="P0"),
  ## attach data
  mxData(observed=data, type='raw'),
  ## maximum likelihood prediction error decomposition estimation
  mxFitFunctionML(),
  mxExpectationStateSpace(x0="x0", P0="P0", u="u", A="Bstar", B="astar", Q="Pstar", C="L", D="ta", R="Th")
)

## get the names of the model parameters and add as CI
name.par <- names(omxGetParameters(model))
model <- mxModel(model, mxCI(reference=name.par))
## run the model
modelout <- mxRun(model, intervals=T)
# print a summary of the results
summary(modelout)
```

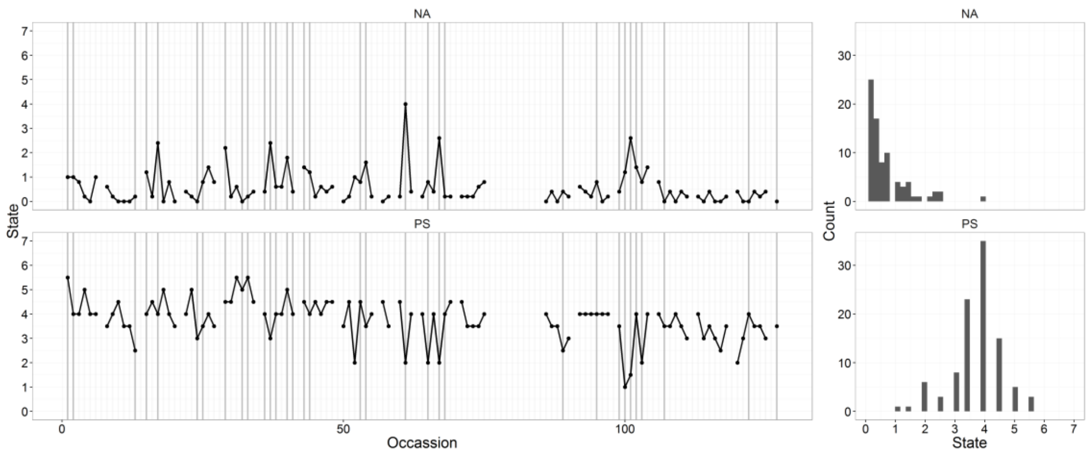
C Negative affect and perceived stress ratings from selected COGITO participants

NA is negative affect, PS is perceived stress, grey vertical background lines indicate the occurrence of at least one relevant negative event before a given occasion on the same day.

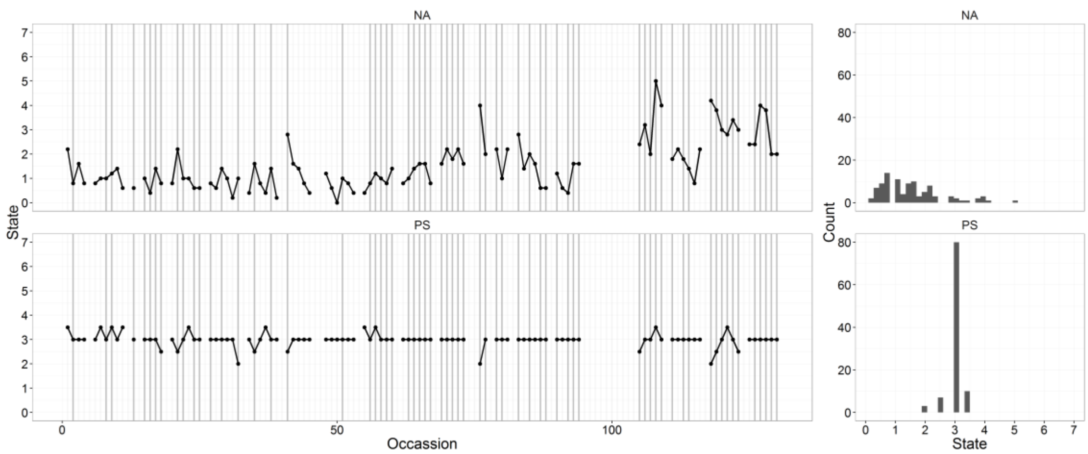
Case 1



Case 2

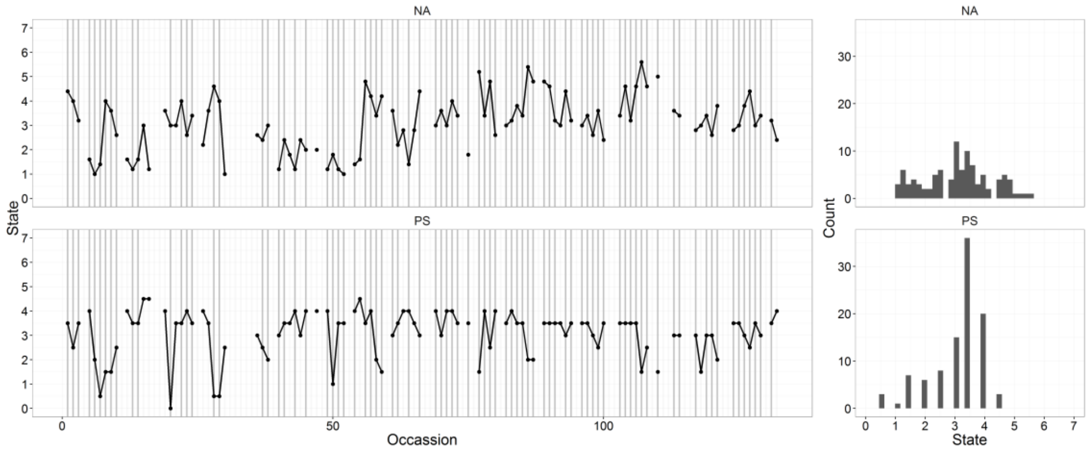


Case 3

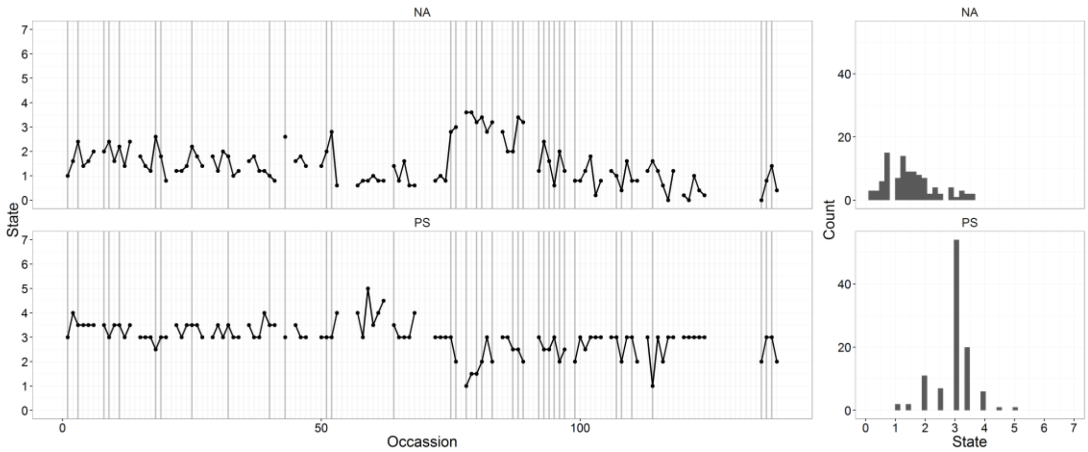


C Negative affect and perceived stress ratings from selected COGITO participants

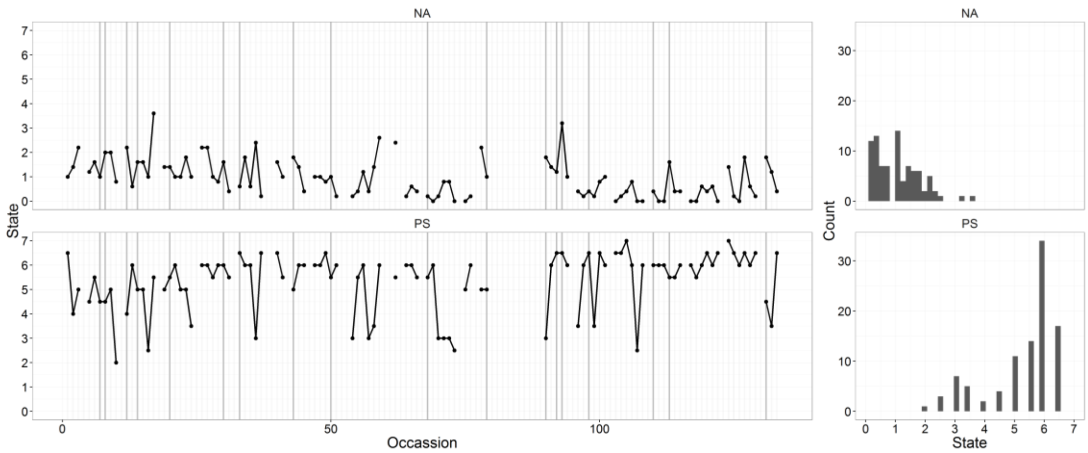
Case 4



Case 5

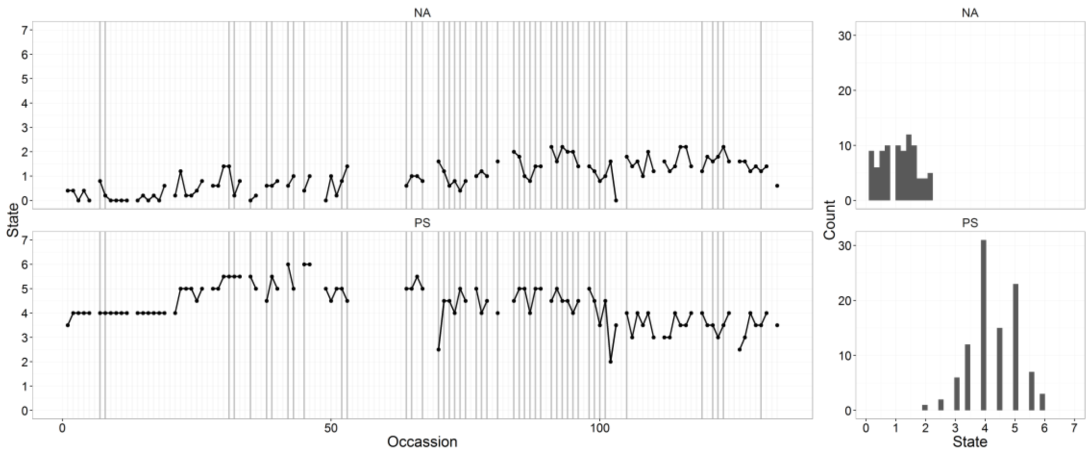


Case 6

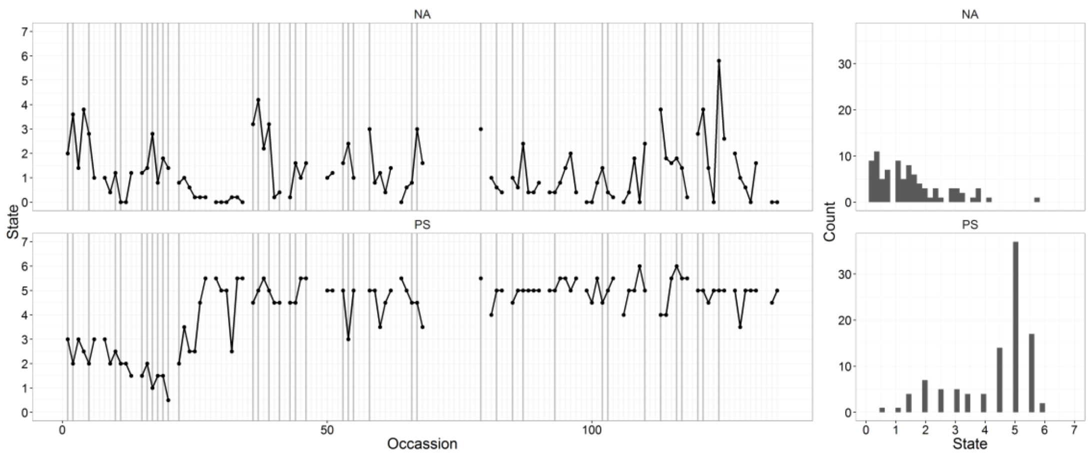


C Negative affect and perceived stress ratings from selected COGITO participants

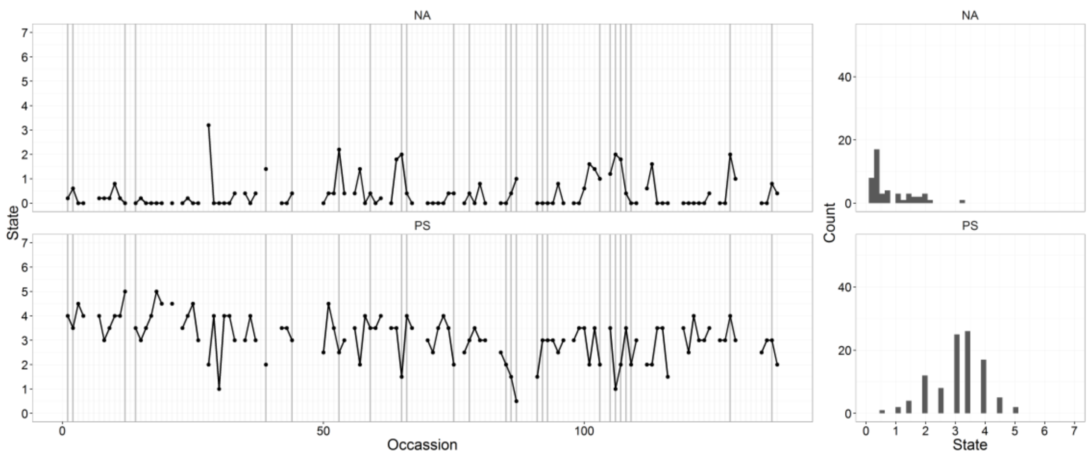
Case 7



Case 8



Case 9



C Negative affect and perceived stress ratings from selected COGITO participants

D Proof of weak stationarity for the latent process of the Poisson autoregressive model

Weak stationarity requires mean stationarity and covariance stationarity to hold. The following proofs are similar to the proofs for the regular AR(1) model (e.g., Brockwell & Davis, 1991, pp. 79–80; Fuller, 1996, pp. 39–40).

We start with proving mean stationarity. Remember that the latent process η_t is defined as follows. For brevity, we drop the index i.

$$\eta_t = \beta\eta_{t-1} + \zeta_t^*, \tag{D.1}$$

where

$$\zeta_t^* \sim lnN(\alpha, \psi)$$

and $|\beta| < 1$ for all t. By repeated substitution, we obtain

$$\begin{aligned} \eta_t &= \beta(\beta\eta_{t-2} + \zeta_{t-1}^*) + \zeta_t^* \\ &= \beta^2\eta_{t-2} + \beta\zeta_{t-1}^* + \zeta_t^* \\ &\vdots \\ &= \beta^h\eta_{t-h} + \sum_{i=0}^{h-1}\beta^i\zeta_{t-i}^* \end{aligned} \tag{D.2}$$

We now show that the above representation can be further reduced to $\eta_t = \sum_{i=0}^{\infty}\beta^i\zeta_{t-i}^*$ in a mean square sense, in that

$$\begin{aligned} \lim_{h\to\infty}\mathbb{E}\left[\left(\eta_t - \sum_{i=0}^{h-1}\beta^i\zeta_{t-i}^*\right)^2\right] &= \lim_{h\to\infty}\mathbb{E}\left[\left(\beta^h\eta_{t-h} + \sum_{i=0}^{h-1}\beta^i\zeta_{t-i}^* - \sum_{i=0}^{h-1}\beta^i\zeta_{t-i}^*\right)^2\right] \\ &= \lim_{h\to\infty}\mathbb{E}[(\beta^h\eta_{t-h})^2] \\ &= \lim_{h\to\infty}\beta^{2h}\mathbb{E}[(\eta_{t-h})^2] \end{aligned} \tag{D.3}$$

If there is any upper bound $K < \infty$ such that $\mathbb{E}[(\eta_{t-h})^2] < K$ for all h, then $\lim_{h\to\infty}\beta^{2h}\mathbb{E}[(\eta_{t-h})^2] = 0$.

We now show that η_t expressed in terms of $\eta_t = \sum_{i=0}^{\infty} \beta^i \zeta_{t-i}^*$ is mean stationary.

$$
\begin{aligned}
\mathbb{E}\left(\lim_{h\to\infty} \sum_{i=0}^{h-1} \beta^i \zeta_{t-i}^*\right) &= \lim_{h\to\infty} \mathbb{E}\left(\sum_{i=0}^{h-1} \beta^i \zeta_{t-i}^*\right) \\
&= \lim_{h\to\infty} \sum_{i=0}^{h-1} \beta^i \mathbb{E}(\zeta_{t-i}^*) \\
&= \lim_{h\to\infty} \sum_{i=0}^{h-1} \beta^i \gamma
\end{aligned}
\tag{D.4}
$$

The first step is valid because of Lemma 2.2.1 in Fuller (p. 31). We know from the ratio criterion that a series $\lim_{h\to\infty} \sum_{i=0}^{h} a_i$ converges if $\lim_{h\to\infty} \frac{a_{i+1}}{a_i} < 1$. For our series, the ratio $\frac{a_{i+1}}{a_i}$ is

$$
\left|\frac{\beta^{i+1}\gamma}{\beta^i \gamma}\right| = |\beta| < 1 \tag{D.5}
$$

for all i. Consequently, the series $\lim_{h\to\infty} \sum_{i=0}^{h} \beta^i \gamma$ converges.

We now proof covariance stationarity. In the previous chapter, we have established that η_t is related to η_{t-h} by

$$
\eta_t = \beta^h \eta_{t-h} + \sum_{i=0}^{h-1} \beta^i \zeta_{t-i}^* \tag{D.6}
$$

The covariance between η_t and η_{t-h} is

$$
\begin{aligned}
Cov(\eta_t, \eta_{t-h}) &= Cov\left(\beta^h \eta_{t-h} + \sum_{i=0}^{h-1} \beta^i \zeta_{t-i}^*, \eta_{t-h}\right) \\
&= Cov(\beta^h \eta_{t-h}, \eta_{t-h}) \\
&= \beta^h Cov(\eta_{t-h}, \eta_{t-h})
\end{aligned}
\tag{D.7}
$$

Thus, the covariance only depends on the distance h.

E Long-run latent process moments for the Poisson autoregressive model

It follows from mean stationarity that a scalar ν exists such that $\mathbb{E}(\eta_t) = \mathbb{E}(\eta_{t-1}) = \nu$ holds. The scalar ν is thus the stationary process mean and equals (recall that the residual process ζ_t^* has mean $\mathbb{E}(\zeta_t^*) = \gamma$):

$$\begin{aligned} \nu &= \mathbb{E}(\beta\eta_{t-1} + \zeta_t^*) \\ &= \beta\mathbb{E}(\eta_{t-1}) + \mathbb{E}(\zeta_t^*) \\ &= \beta\nu + \gamma \\ \Leftrightarrow \gamma &= \nu - \beta\nu = (1-\beta)\nu \\ \Leftrightarrow \nu &= \frac{\gamma}{1-\beta} \end{aligned} \tag{E.1}$$

It follows from covariance stationarity that a scalar ρ exists such that $\mathbb{E}((\eta_t - \mathbb{E}(\eta_t))^2) = \mathbb{E}((\eta_t)^2) - \nu^2 = \mathbb{E}((\eta_{t-1})^2) - \nu^2 = \rho$. The scalar ρ is thus the stationary process variance and equals (recall that the residual process ζ_t^* has variance $\mathbb{E}((\zeta_t^*)^2) - \gamma^2 = \omega$):

$$
\begin{aligned}
\rho &= \mathbb{E}((\beta\eta_{t-1} + \zeta_t^*)^2\,) - \nu^2 \\
&= \mathbb{E}((\beta\eta_{t-1})^2 + (\zeta_t^*)^2 + 2\beta\eta_{t-1}\zeta_t^*) - \nu^2 \\
&= \beta^2\mathbb{E}((\eta_{t-1})^2) + \mathbb{E}((\zeta_t^*)^2) + 2\beta\mathbb{E}(\eta_{t-1}\zeta_t^*) - \nu^2 \\
&= \beta^2\rho + \beta^2\nu^2 + \omega + \gamma^2 + 2\beta\nu\gamma - \nu^2 \\
&= \beta^2\rho + \omega + (\beta^2\nu^2 + \gamma^2 + 2\beta\nu\gamma) - \nu^2 \\
&= \beta^2\rho + \omega + \left(\beta^2\left(\frac{\gamma}{1-\beta}\right)^2 + \gamma^2 + 2\beta\left(\frac{\gamma}{1-\beta}\right)\gamma\right) - \nu^2 \\
&= \beta^2\rho + \omega + \left(\frac{\beta^2\gamma^2}{(1-\beta)^2} + \gamma^2 + \frac{2\beta\gamma^2}{1-\beta}\right) - \nu^2 \\
&= \beta^2\rho + \omega + \left(\frac{\beta^2\gamma^2}{(1-\beta)^2} + \frac{\gamma^2(1-\beta)^2}{(1-\beta)^2} + \frac{2\beta\gamma^2(1-\beta)}{(1-\beta)^2}\right) - \nu^2 \\
&= \beta^2\rho + \omega + \left(\frac{\beta^2\gamma^2 + \gamma^2 - 2\beta\gamma^2 + \beta^2\gamma^2 + 2\beta\gamma^2 - 2\beta^2\gamma^2}{(1-\beta)^2}\right) - \nu^2 \\
&= \beta^2\rho + \omega + \left(\frac{\gamma^2(\beta^2 + 1 - 2\beta + \beta^2 + 2\beta - 2\beta^2)}{(1-\beta)^2}\right) - \nu^2 \\
&= \beta^2\rho + \omega + \frac{\gamma^2}{(1-\beta)^2} - \nu^2 \\
&= \beta^2\rho + \omega \\
\Leftrightarrow \omega &= \rho - \beta^2\rho = (1-\beta^2)\rho \\
\Leftrightarrow \rho &= \frac{\omega}{1-\beta^2}
\end{aligned}
\tag{E.2}
$$

F Long-run latent process moments for the hybrid Poisson-Gaussian vector autoregressive model

It follows from mean stationarity that a vector $\boldsymbol{\nu}$ exists such that $\mathbb{E}(\boldsymbol{\eta}_{t-1}) = \mathbb{E}(\boldsymbol{\eta}_t) = \boldsymbol{\nu}$. The vector $\boldsymbol{\nu}$ is thus the stationary process mean and equals (recall that the residual process $\boldsymbol{\zeta}_t^*$ has mean vector $\mathbb{E}(\boldsymbol{\zeta}_t^*) = \boldsymbol{\gamma}$):

$$
\begin{aligned}
\boldsymbol{\nu} &= \mathbb{E}(\mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t^*) \\
&= \mathbf{B}\mathbb{E}(\boldsymbol{\eta}_{t-1}) + \mathbb{E}(\boldsymbol{\zeta}_t^*) \\
&= \mathbf{B}\boldsymbol{\nu} + \boldsymbol{\gamma} \\
\Leftrightarrow \boldsymbol{\gamma} &= \boldsymbol{\nu} - \mathbf{B}\boldsymbol{\nu} = (\mathbf{I} - \mathbf{B})\boldsymbol{\nu} \\
\Leftrightarrow \boldsymbol{\nu} &= (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\gamma},
\end{aligned} \tag{F.1}
$$

It follows from covariance stationarity that a matrix $\mathbf{P}$ exists such that $\mathbb{E}\left(\left(\boldsymbol{\eta}_t - \mathbb{E}(\boldsymbol{\eta}_t)\right)\left(\boldsymbol{\eta}_t - \mathbb{E}(\boldsymbol{\eta}_t)\right)^{\mathrm{T}}\right) = \mathbb{E}\left(\boldsymbol{\eta}_t\boldsymbol{\eta}_t^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} = \mathbb{E}\left(\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} = \mathbf{P}$. The matrix $\mathbf{P}$ is thus the stationary process covariance matrix and equals (recall that the residual process $\boldsymbol{\zeta}_t^*$ has covariance matrix $\mathbb{E}\left(\boldsymbol{\zeta}_t^*\boldsymbol{\zeta}_t^{*\mathrm{T}}\right) - \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}} = \boldsymbol{\Omega}$):

$$
\begin{aligned}
\mathbf{P} &= \mathbb{E}\left((\mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t^*)(\mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t^*)^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbb{E}\left((\mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t^*)\left(\boldsymbol{\eta}_{t-1}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \boldsymbol{\zeta}_t^{*\mathrm{T}}\right)\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbb{E}\left(\mathbf{B}\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \boldsymbol{\zeta}_t^*\boldsymbol{\zeta}_t^{*\mathrm{T}} + \boldsymbol{\zeta}_t^*\boldsymbol{\eta}_{t-1}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\boldsymbol{\eta}_{t-1}\boldsymbol{\zeta}_t^{*\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{B}\mathbb{E}(\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathrm{T}})\mathbf{B}^{\mathrm{T}} + \mathbb{E}\left(\boldsymbol{\zeta}_t^*\boldsymbol{\zeta}_t^{*\mathrm{T}}\right) + \mathbb{E}(\boldsymbol{\zeta}_t^*\boldsymbol{\eta}_{t-1}^{\mathrm{T}})\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbb{E}(\boldsymbol{\eta}_{t-1}\boldsymbol{\zeta}_t^{*\mathrm{T}}) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \mathbf{B}\boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \boldsymbol{\Omega} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}} + \boldsymbol{\gamma}\boldsymbol{\nu}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\boldsymbol{\nu}\boldsymbol{\gamma}^{\mathrm{T}} - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + (\mathbf{B}\boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}} + \boldsymbol{\gamma}\boldsymbol{\nu}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\boldsymbol{\nu}\boldsymbol{\gamma}^{\mathrm{T}}) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left(\mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}} + \boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left(\mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}^{-1}(\mathbf{I}-\mathbf{B})\mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}(\mathbf{I}-\mathbf{B})^{\mathrm{T}}{\mathbf{B}^{-1}}^{\mathrm{T}}\right. \\
&\quad \left. + \mathbf{B}^{-1}(\mathbf{I}-\mathbf{B})\mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}(\mathbf{I}-\mathbf{B})^{\mathrm{T}}{\mathbf{B}^{-1}}^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left((\mathbf{B} + \mathbf{B}^{-1}(\mathbf{I}-\mathbf{B})\mathbf{B})(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\right. \\
&\quad \left. + (\mathbf{B}^{-1}(\mathbf{I}-\mathbf{B})\mathbf{B} + \mathbf{B})(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}(\mathbf{I}-\mathbf{B})^{\mathrm{T}}{\mathbf{B}^{-1}}^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left(\mathbf{I}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{I}(\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}(\mathbf{I}-\mathbf{B})^{\mathrm{T}}{\mathbf{B}^{-1}}^{\mathrm{T}}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left((\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\left(\mathbf{B}^{\mathrm{T}} + \mathbf{B}^{\mathrm{T}}(\mathbf{I}-\mathbf{B})^{\mathrm{T}}{\mathbf{B}^{-1}}^{\mathrm{T}}\right)\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \left((\mathbf{I}-\mathbf{B})^{-1}\boldsymbol{\gamma}\boldsymbol{\gamma}^{\mathrm{T}}{(\mathbf{I}-\mathbf{B})^{-1}}^{\mathrm{T}}\mathbf{I}\right) - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} + \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} - \boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{T}} \\
&= \mathbf{BPB}^{\mathrm{T}} + \boldsymbol{\Omega} \\
\Leftrightarrow vec(\mathbf{P}) &= (\mathbf{B} \otimes \mathbf{B})vec(\mathbf{P}) + vec(\boldsymbol{\Omega}) \\
\Leftrightarrow vec(\boldsymbol{\Omega}) &= vec(\mathbf{P}) - (\mathbf{B} \otimes \mathbf{B})vec(\mathbf{P}) = \left(\mathbf{I} - (\mathbf{B} \otimes \mathbf{B})\right)vec(\mathbf{P}) \\
\Leftrightarrow vec(\mathbf{P}) &= \left(\mathbf{I} - (\mathbf{B} \otimes \mathbf{B})\right)^{-1} vec(\boldsymbol{\Omega}).
\end{aligned}
\tag{F.2}
$$

G JAGS script for the hybrid Poisson-Gaussian vector autoregressive model

```
model {
for (i in 1:N) {
for (t in 2:T) {
#
# LEVEL IA
#
zetacom[t,i] ~ dnorm(null[1], I[1,1])
y[t,1,i] ~ dpois(eta[t,1,i])
y[t,2,i] ~ dnorm(eta[t,2,i], epsilon.prec)
for (k in 1:2){
zeta[t,k,i] ~ dnorm(zetaload[k] * zetacom[t,i],zetares.prec)
eta[t,k,i] <- beta[k,1] * eta[t-1,1,i] + beta[k,2] * eta[t-1,2,i] + exp(alpha[k,i] + zeta[t,k,i])
}
} # end T
# initialization at t0
eta[1,1,i] <- eta.help[1,i] ; eta[1,2,i] <- eta.help[2,i]
eta.help[1:2,i] <- Inv.ImB %*% gamma.est[1:2,i]
gamma.est[1,i] <- exp(alpha[1,i] + psi[1,1]/2); gamma.est[2,i] <- exp(alpha[2,i] + psi[2,2]/2)
#
# LEVEL IE
#
for (k in 1:2) { alpha[k,i] <- kappa[k] + ksi[k,i] }
ksi[1:2,i] ~ dmnorm(null, ksi.prec)
} # end N
#
# PRIORS
#
# random intercepts
for (k in 1:2){ kappa[k] ~ dnorm(0, .01)}
ksi.prec ~ dwish(I, 2)
phi <- inverse(ksi.prec)
#
# fixed autoregressive effects
beta[1,1] ~ dnorm(0,.01); beta[2,2] ~ dnorm(0,.01)
beta[1,2] ~ dnorm(0,.01); beta[2,1] ~ dnorm(0,.01)
#
# fixed process error variance
for (k in 1:2){ zetaload[k] ~ dnorm(0, .01) }
```

```
zetares.prec ~ dgamma(.001 ,.001)
psi[1,1] <- zetaload[1]^2 + zetares.prec^(-1); psi[2,2] <- zetaload[2]^2 + zetares.prec^(-1)
psi[1,2] <- zetaload[1]*zetaload[2]; psi[2,1] <- zetaload[1]*zetaload[2]
#
# fixed measurement error variance
epsilon.prec ~ dgamma(.001 ,.001)
theta <- pow(epsilon.prec, -1)
#
# HELPERS
#
# constants
null[1]<-0; null[2]<-0
I[1,1]<-1; I[2,1]<-0; I[1,2]<-0; I[2,2]<-1
#
# inverse of (I - beta)
ImB <- I - beta
ImB.det <- ImB[1,1]*ImB[2,2] - ImB[2,1]*ImB[1,2] # determinant
ImB.invertible <- ifelse( ImB.det == 0, 0, 1 )  # ImB invertible?
ImB.detmod1 ~ dbern(0.5) # if not invertible, modest modification of determinant
ImB.detmod2 <- ifelse(ImB.invertible == 1, 0, (ImB.detmod1-0.5)/5000)
ImB.det.adj <- ImB.det + ImB.detmod2
ImB.help[1,1] <- ImB[2,2]; ImB.help[2,1] <- ImB[2,1]*-1; ImB.help[1,2] <- ImB[1,2]*-1; ImB.help[2,2] <- ImB[1,1]
Inv.ImB <- 1/ImB.det.adj * ImB.help
}
```

H Models fitted to COGITO data – traceplots and marginal posterior distributions

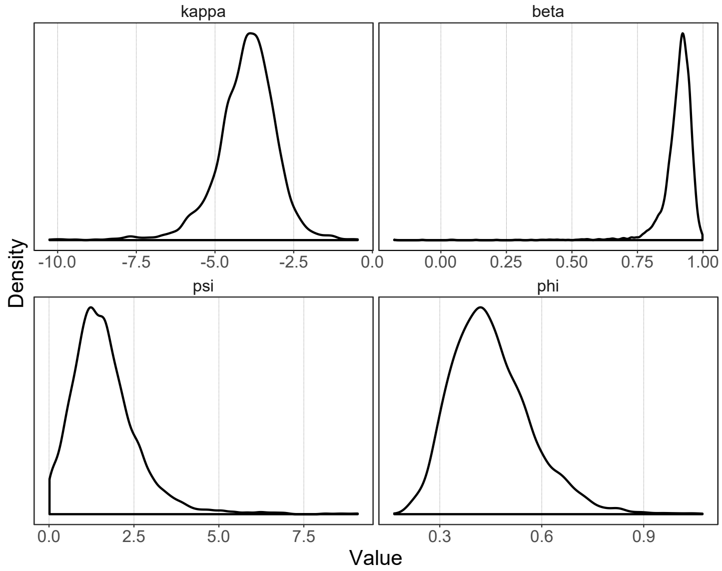


Figure H.1. Marginal posterior distributions for the PAR model fitted to COGITO event data.

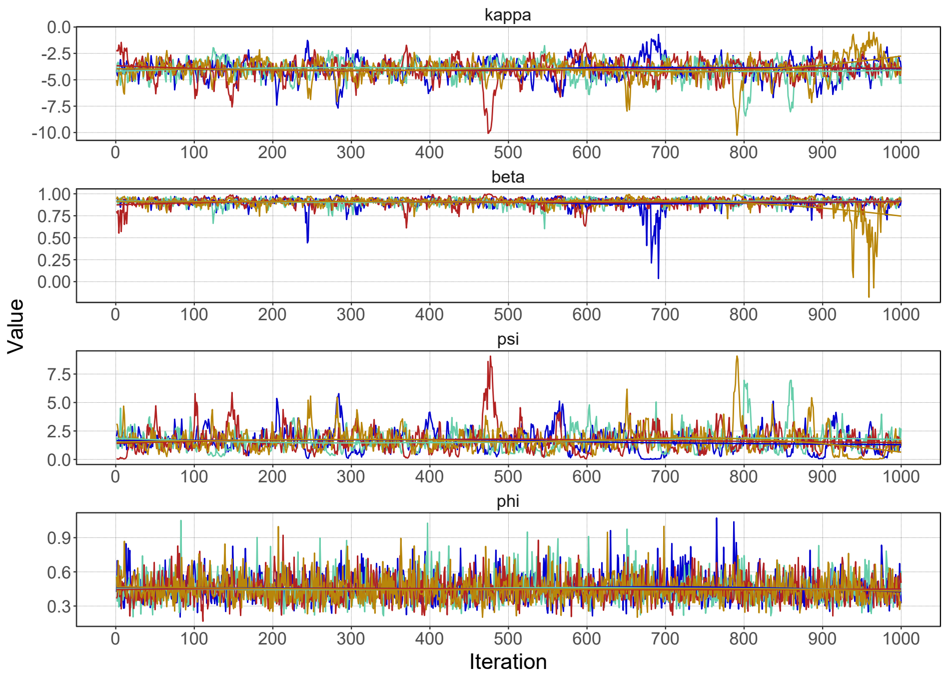


Figure H.2. Traceplots for PAR model fitted to COGITO event data.

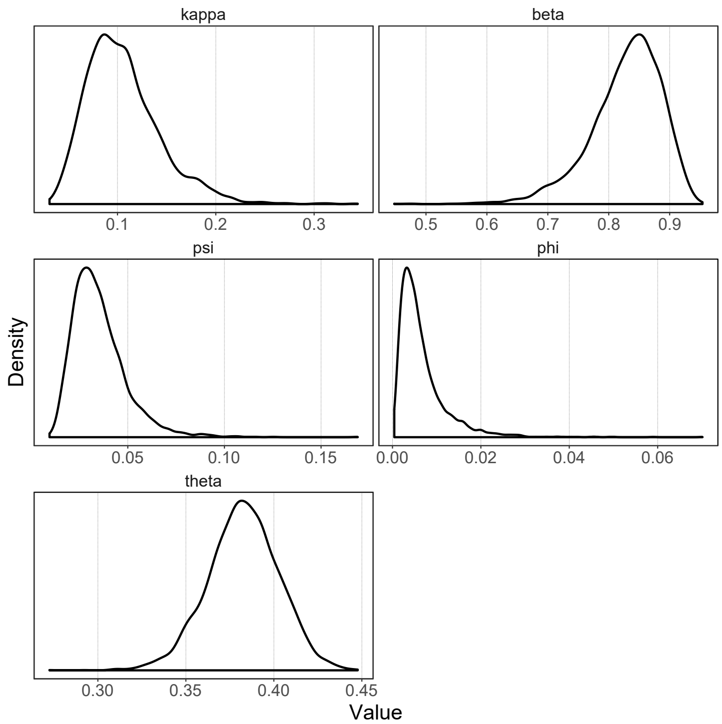


Figure H.3. Marginal posterior distributions for the AR model fitted to COGITO event data.

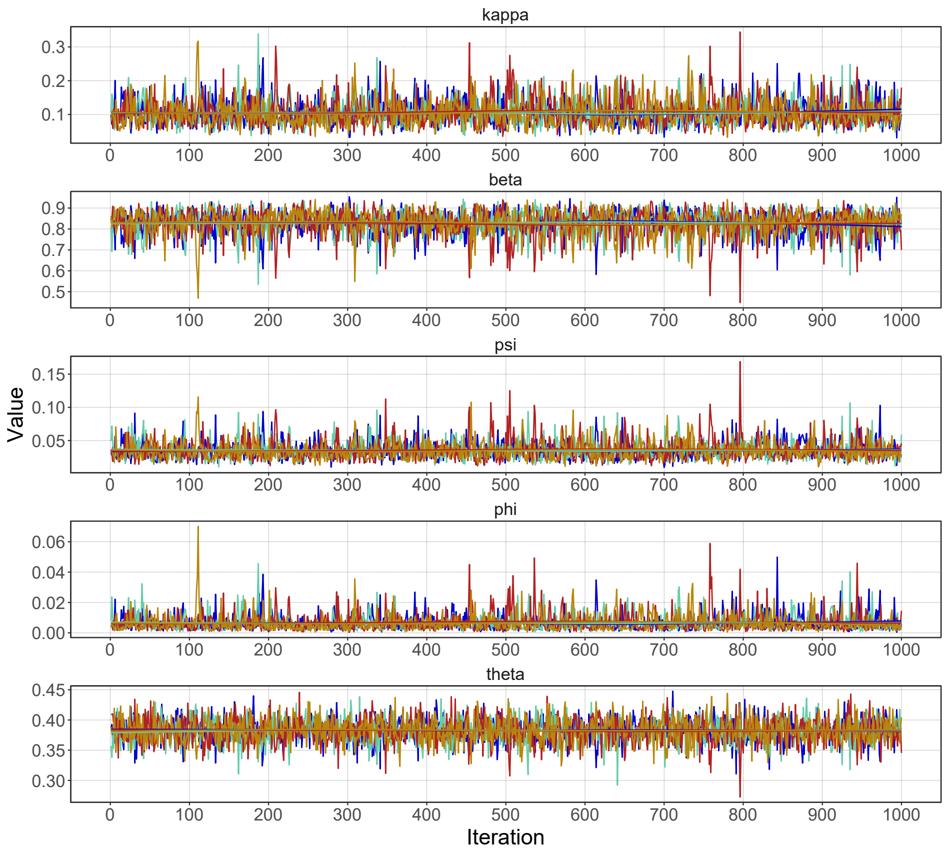


Figure H.4. Traceplots for AR model fitted to COGITO event data.

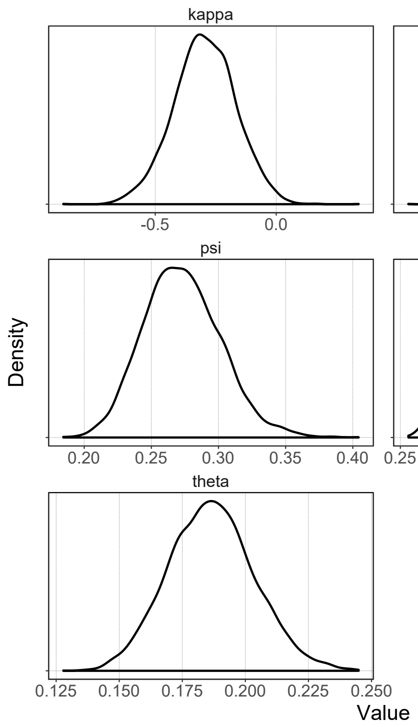


Figure H.5. Marginal posterior distributions for the AR model with log link fitted to COGITO affect data.

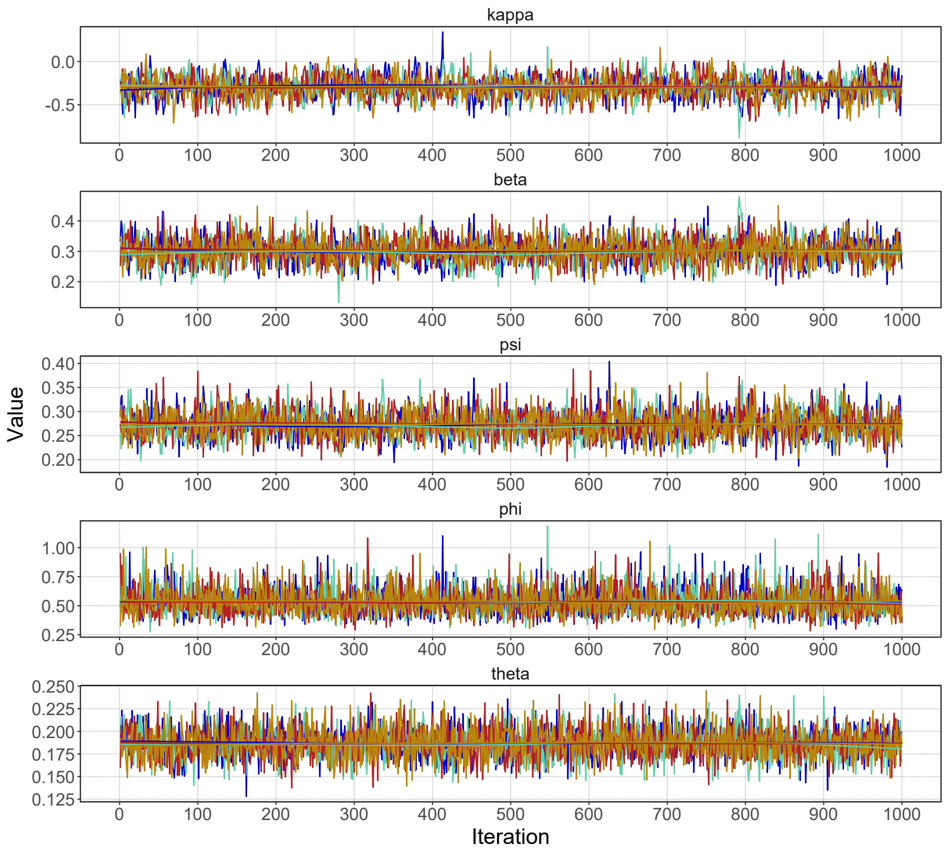


Figure H.6. Traceplots for the AR model with log link fitted to COGITO affect data.

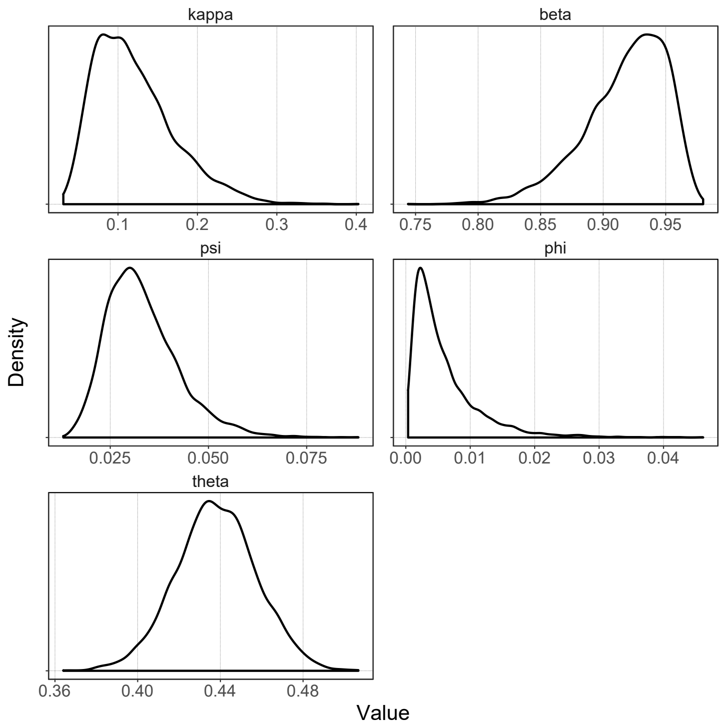


Figure H.7. Marginal posterior distributions for the AR model fitted to COGITO affect data.

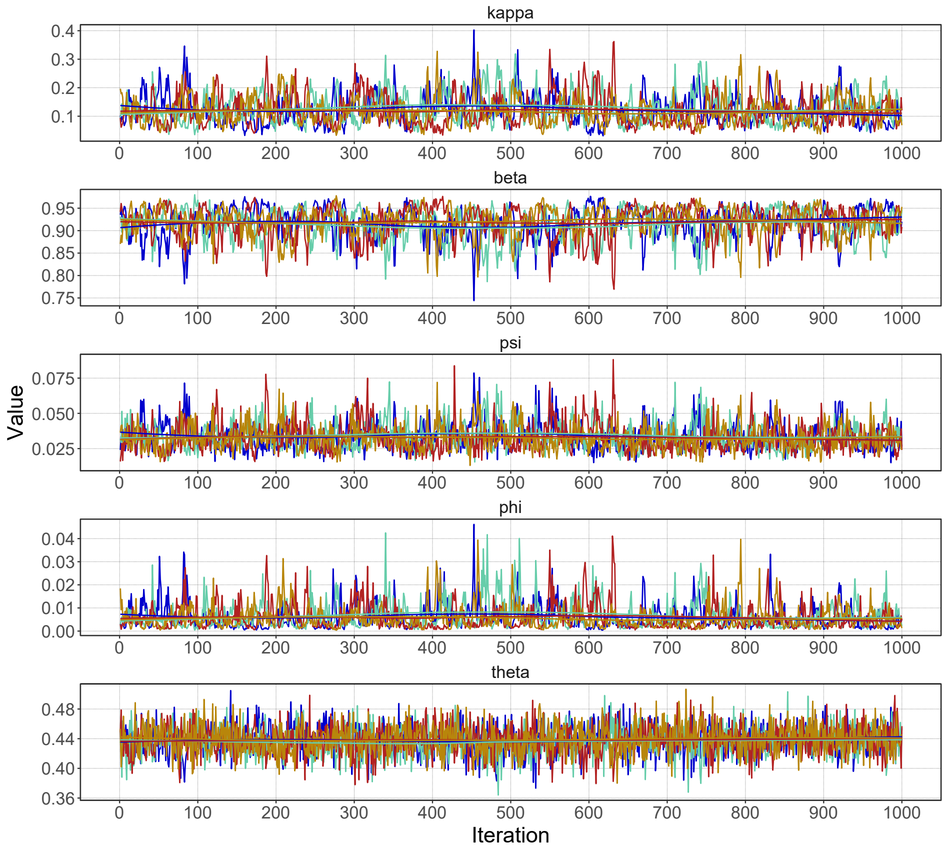


Figure H.8. Traceplots for the AR model fitted to COGITO affect data.

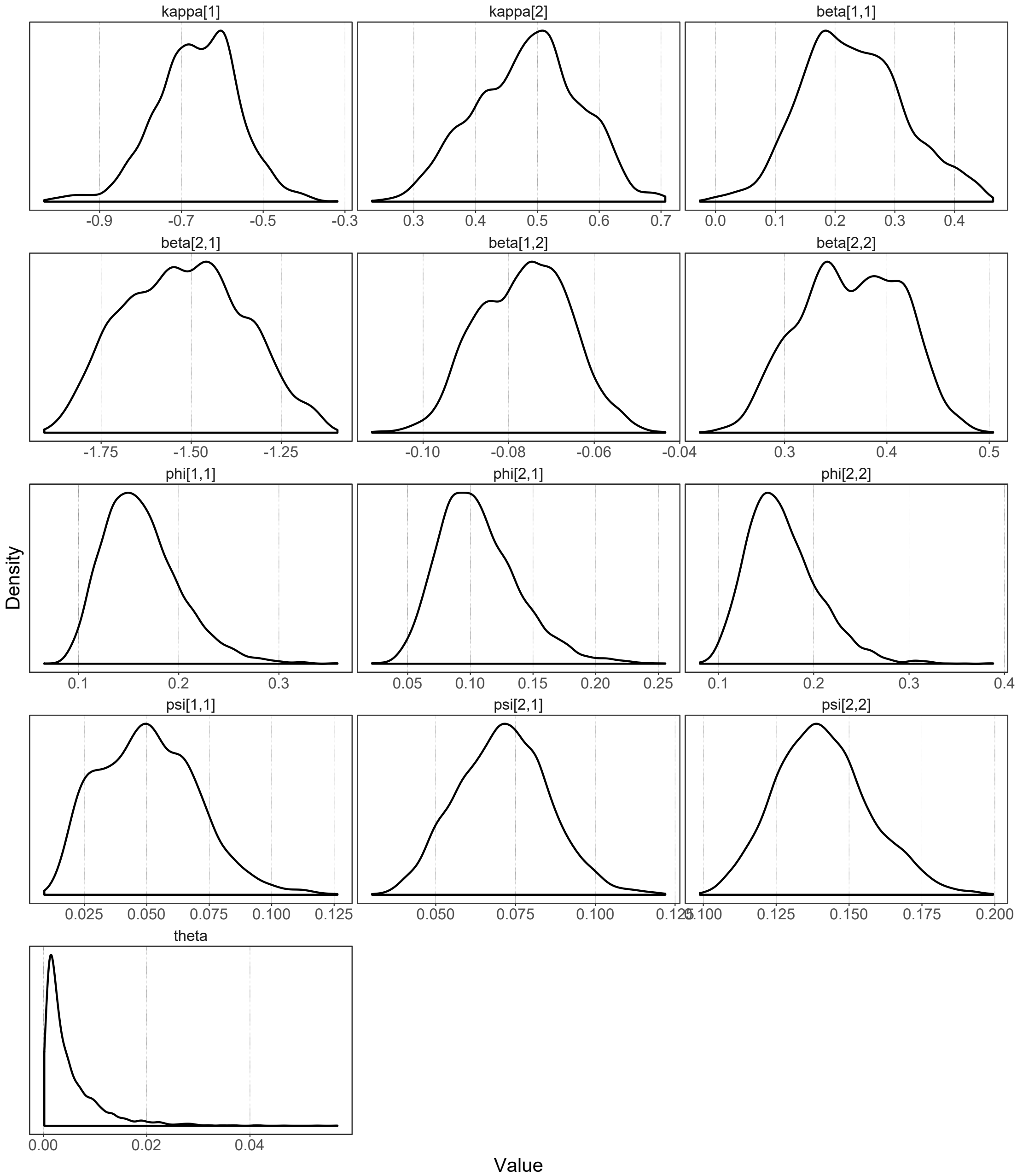


Figure H.9. Marginal posterior distributions for the hPVAR model fitted to COGITO event and affect data.

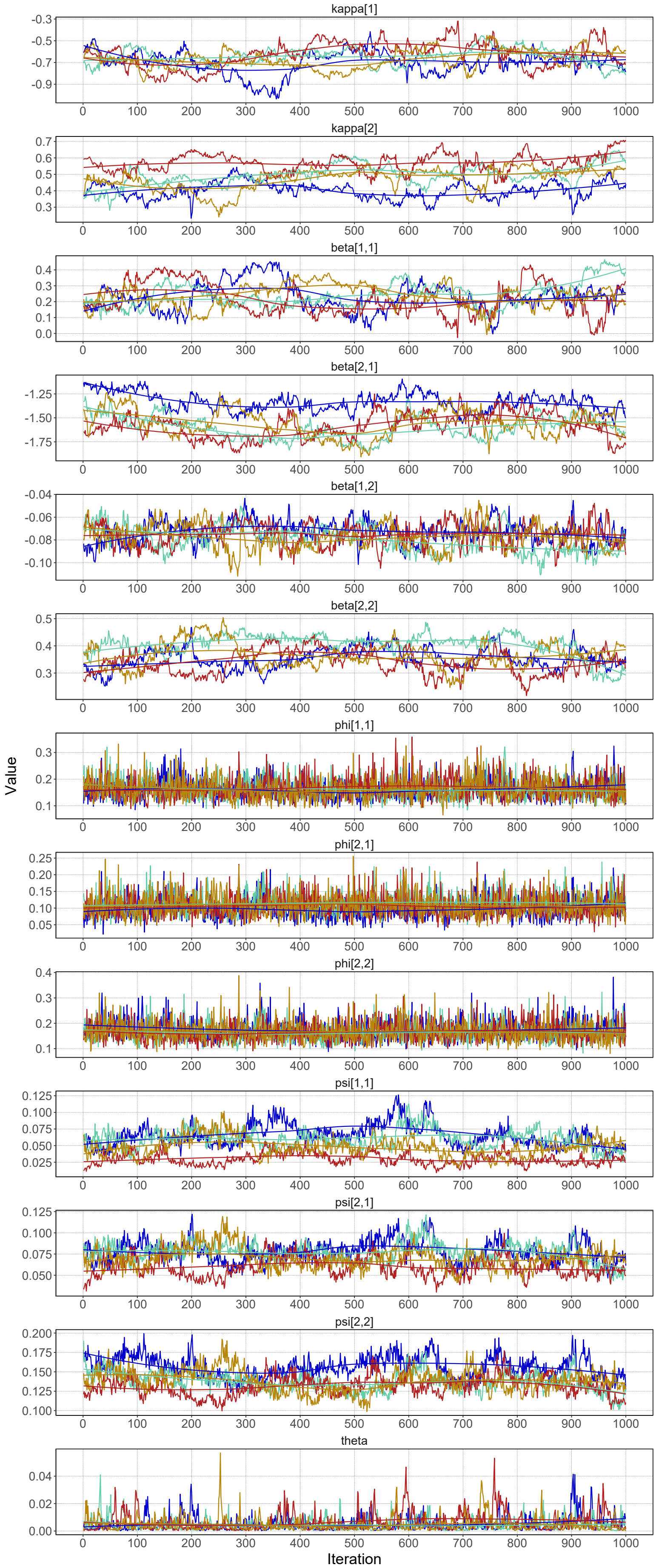


Figure H.10. Traceplots for the hPVAR model fitted to COGITO event and affect data.

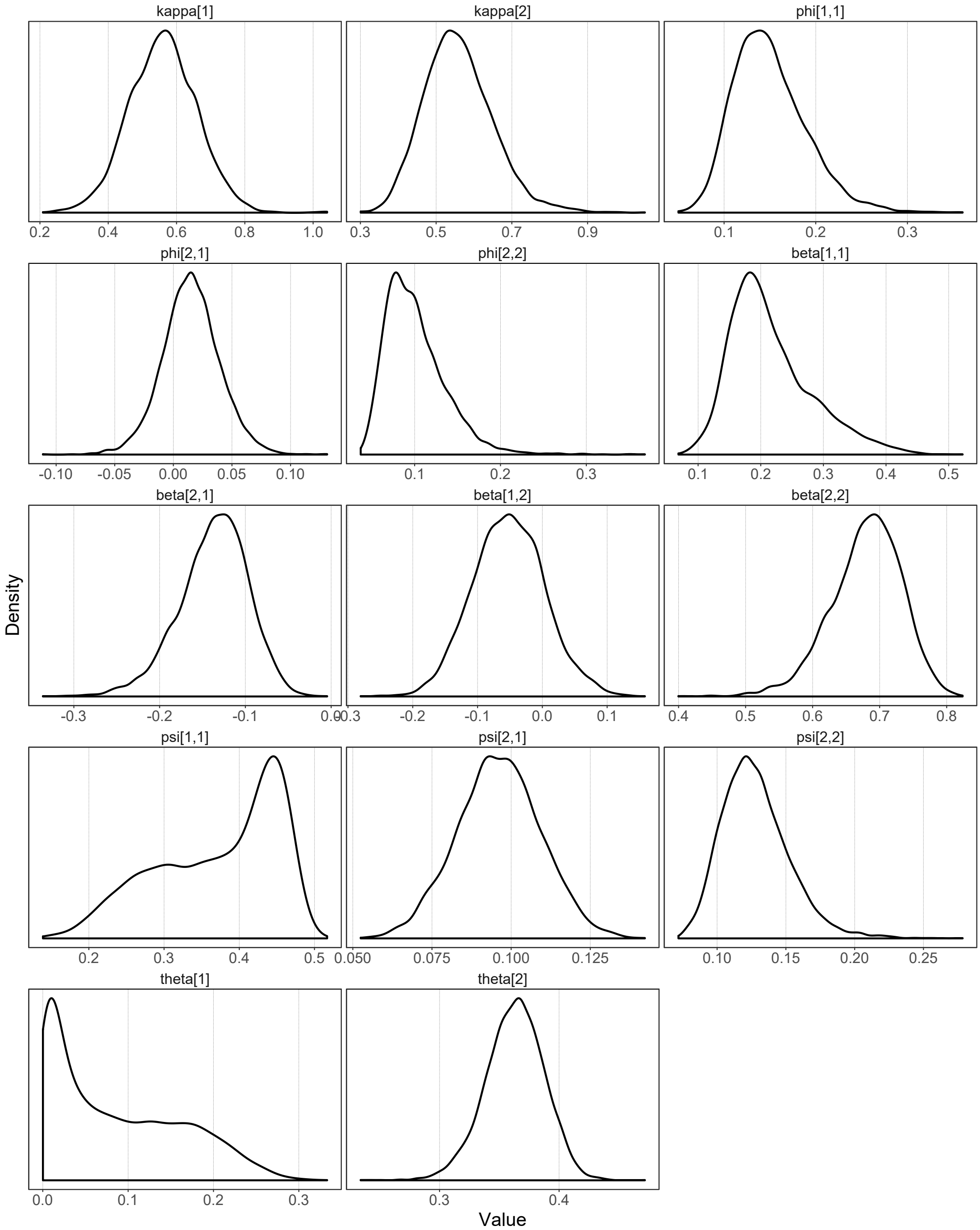


Figure H.11. Marginal posterior distributions for the VAR model fitted to event and affect data.

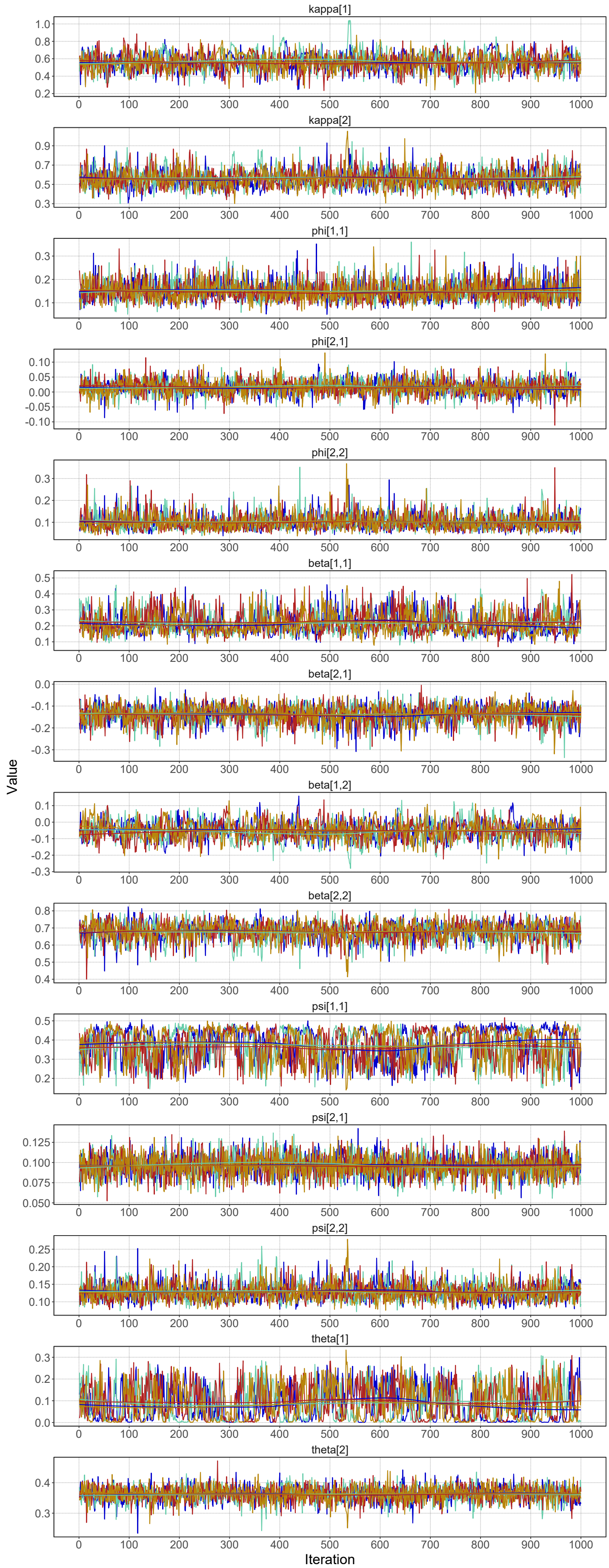


Figure H.12. Traceplots for the VAR model fitted to COGITO event and affect data.